Chapter 1

$$\bar{d}q = d\mathcal{E} + dW \qquad dq = de + p\,d\left(\frac{1}{\rho}\right) = de - \frac{p}{\rho^2}\,d\rho$$

$$de = \left.\frac{\partial e}{\partial T}\right|_p dT + \left.\frac{\partial e}{\partial p}\right|_T dp$$

$$d\rho = \left.\frac{\partial \rho}{\partial T}\right|_p dT + \left(\frac{\partial \rho}{\partial p}\right)_T dp$$

2.25-2.27

$$dq = \left.\frac{\partial e}{\partial T}\right|_p dT + \left.\frac{\partial e}{\partial p}\right|_T dp - \frac{p}{\rho^2}\left(\left.\frac{\partial \rho}{\partial T}\right|_p dT + \left.\frac{\partial \rho}{\partial p}\right|_T dp\right)$$

$$= \left\{\left.\frac{\partial e}{\partial T}\right|_p - \frac{p}{\rho^2}\left.\frac{\partial \rho}{\partial T}\right|_p\right\} dT + \left\{\left.\frac{\partial e}{\partial p}\right|_T - \frac{p}{\rho^2}\left.\frac{\partial \rho}{\partial p}\right|_T\right\} dp$$

$$\frac{C_V - C_P}{\left(\partial p/\partial T\right)_\rho} = \frac{C_P - C_V}{1\Big/\left.\frac{\partial T}{\partial \rho}\right|_p \cdot \left.\frac{\partial \rho}{\partial p}\right|_T} = \frac{C_P - C_V}{\underset{(-\beta)}{\left(\left.\frac{\partial \ln\rho}{\partial T}\right|_P\right)} \Big/ \underset{(+\cancel{K_T})}{\left(\left.\frac{\partial \ln\rho}{\partial P}\right|_{\cancel{P}}\right)}}$$

$$-\frac{K_T}{\beta}(C_P - C_V) = \left.\frac{\partial e}{\partial p}\right|_T - \frac{p}{\rho}\left.\frac{\partial \ln\rho}{\partial p}\right|_T$$

$$\left.\frac{\partial e}{\partial p}\right|_T = \frac{p}{\rho}\left.\frac{\partial \ln\rho}{\partial p}\right|_T - \frac{K_T}{\beta}(C_P - C_V)$$

$$= \frac{p}{\rho}K_T - \frac{K_T(C_P - C_V)}{\beta} = K_T\left(\frac{p}{\rho} - \frac{(C_P - C_V)}{\beta}\right)$$

2.28-2.30

$$dq = \frac{\partial e}{\partial p}\bigg|_{\rho} dp + \left(\frac{\partial e}{\partial \rho}\bigg|_{p} - \frac{p}{\rho^2}\right) d\rho$$

$$= \frac{\partial e}{\partial p}\bigg|_{\rho} dp + \left(\frac{\partial e}{\partial \rho}\bigg|_{p} - \frac{p}{\rho^2}\right)\left(\frac{\partial \rho}{\partial p}\bigg|_{T} dp + \frac{\partial \rho}{\partial T}\bigg|_{p} dT\right)$$

$$c_p\, dT = \left(\frac{\partial e}{\partial \rho}\bigg|_{p} - \frac{p}{\rho^2}\right)\left(\frac{\partial \rho}{\partial T}\bigg|_{p}\right) dT$$

$$= \left(\frac{\partial e}{\partial \rho}\bigg|_{p} - \frac{p}{\rho^2}\right)\rho(-\beta)\, dT$$

$$-\frac{c_p}{\beta\rho} = \frac{\partial e}{\partial \rho}\bigg|_{p} - \frac{p}{\rho^2}$$

$$\frac{p}{\rho^2} - \frac{c_p}{\beta\rho} = \frac{\partial e}{\partial \rho}\bigg|_{p}$$

$$\frac{dp}{d\rho} = -\frac{\left(\frac{\partial e}{\partial \rho}\Big|_{p} - \frac{p}{\rho^2}\right)}{\left(\frac{\partial e}{\partial p}\Big|_{\rho}\right)}$$

$$= \frac{+\,c_p/\cancel{\beta}\rho}{k_T c_v/\cancel{\beta}}$$

$$= \frac{c_p}{\rho k_T c_v}$$

5.5

$$\frac{1}{T}\cancel{\frac{\partial^2 e}{\partial\rho\,\partial T}} = -\frac{1}{T}\left[\left.\frac{\partial e}{\partial\rho}\right|_T - \frac{p}{\rho^2}\right] + \frac{1}{T}\cancel{\frac{\partial^2 e}{\partial T\,\partial\rho}} - \left.\frac{1}{\rho^2}\frac{\partial p}{\partial T}\right|_\rho$$

$$-\frac{1}{T}\left.\frac{\partial e}{\partial\rho}\right|_T + \frac{p}{\rho^2 T} - \left.\frac{1}{\rho^2}\frac{\partial p}{\partial T}\right|_\rho = 0$$

$$\frac{1}{T}\left.\frac{\partial e}{\partial\rho}\right|_T = \frac{p}{\rho^2 T} - \left.\frac{1}{\rho^2}\frac{\partial p}{\partial T}\right|_\rho$$

$$\left.\frac{\partial e}{\partial\rho}\right|_T = \frac{p}{\rho^2} - \left.\frac{T}{\rho^2}\frac{\partial p}{\partial T}\right|_\rho$$

$p = \rho R T$

$\left.\frac{\partial p}{\partial T}\right| = \rho R$

$p - T\cdot\rho R \equiv 0$ for perfect gas

$$\left.\frac{\partial p}{\partial T}\right|_\rho \cdot \left.\frac{\partial T}{\partial\rho}\right|_p \cdot \left.\frac{\partial\rho}{\partial p}\right|_T = -1$$

$$\left.\frac{\partial p}{\partial T}\right|_\rho \left(\left.\frac{\partial \ln\rho}{\partial p}\right|_T \Big/ \left.\frac{\partial \ln\rho}{\partial T}\right|_p\right) = -1$$

$$\left.\frac{\partial p}{\partial T}\right|_\rho \left(K_T \big/ (-\beta)\right) = -1$$

$$\left.\frac{\partial p}{\partial T}\right|_\rho = \frac{\beta}{K_T}$$

5.7

$$\left.\frac{\partial e}{\partial \rho}\right|_T = \frac{1}{\rho^2}\left[p - \frac{\beta T}{K_T}\right]$$

$$c_p - c_v = \left[\left.\frac{\partial e}{\partial \rho}\right|_T - \frac{p}{\rho^2}\right]\left.\frac{\partial \rho}{\partial T}\right|_T$$

$$= \left\{\frac{1}{\rho^2}\left(p - \frac{\beta T}{K_T}\right) - \frac{p}{\rho^2}\right\}\left.\frac{\partial \rho}{\partial T}\right|_T$$

$$c_p - c_v = -\frac{\beta T}{\rho K_T}\left.\frac{\partial \ln \rho}{\partial T}\right|_T$$

$$c_p - c_v = \left(\left.\frac{\partial e}{\partial \rho}\right|_T - \frac{p}{\rho^2}\right)\left.\frac{\partial \rho}{\partial T}\right|_p$$

$$\left.\frac{\partial e}{\partial \rho}\right|_T = \frac{1}{\rho^2}\left(p - \frac{\beta T}{K_T}\right)$$

$$c_p - c_v = \left(\frac{\cancel{p}}{\cancel{\rho}} - \frac{\beta T}{\rho^2 K_T} - \frac{\cancel{p}}{\cancel{\rho^2}}\right)\left.\frac{\partial \rho}{\partial T}\right|_p$$

$$c_p - c_v = -\frac{\beta T}{\rho^2 K_T}\boxed{\left.\frac{\partial \rho}{\partial T}\right|_p} \equiv -\beta$$

$$= +\frac{\beta^2 T}{\rho K_T}$$

$\frac{\partial s}{\partial T}\Big|_\rho$

5.8-5.10

$$\frac{1}{T}\frac{\partial e}{\partial T}\Big|_\rho = \frac{\partial s}{\partial T}\Big|_\rho$$

c_V

$$d\rho = \frac{\partial \rho}{\partial T}\Big|_p dp + \frac{\partial \rho}{\partial p}\Big|_T$$

$$\cancel{c_p dT = \frac{\partial e}{\partial p}\Big|}$$

$$c_p dT = \frac{\partial e}{\partial T}\Big|_p dT + \cancel{\frac{\partial e}{\partial T}\Big|_p} dp - \frac{p}{\rho^2}\left(\cancel{dp}\frac{\partial \rho}{\partial T}\Big|_p dp + \frac{\partial \rho}{\partial p}\Big|_T dT\right)$$

$$c_p = \frac{\partial e}{\partial T}\Big|_p - \frac{p}{\rho^2}\cdot\frac{\partial \rho}{\partial p}\Big|_T$$

$$\frac{\partial \rho}{\partial T}\Big|_p dT + \frac{\partial \rho}{\partial p}\Big|_T dp$$

$$\frac{\partial s}{\partial T}\Big|_\rho = \frac{1}{T}\frac{\partial e}{\partial T}\Big|_\rho$$

$$\frac{\partial e}{\partial T}\Big|_\rho \cdot \boxed{\frac{\partial T}{\partial \rho}\Big|_e} \cdot \frac{\partial \rho}{\partial e}\Big|_T$$

$$ds = \frac{\partial s}{\partial T}\Big|_\rho dT + \frac{\partial s}{\partial \rho}\Big|_T d\rho$$

$$= \frac{de}{\partial \rho}\Big|_T d\rho + \frac{de}{\partial T}\Big|_\rho dT - \frac{p}{\rho^2} d\rho$$

5.9 $$\left.\frac{\partial s}{\partial \rho}\right|_T = \frac{1}{T\cancel{\rho}}\left\{\frac{1}{\rho^2}\left(p - \frac{\beta T}{K_T}\right) - \frac{p}{\rho^2}\right\}$$

$$= \frac{1}{T}\left\{-\frac{\beta T}{\rho^2 K_T}\right\}$$

$$= -\frac{\beta}{\rho^2 K_T}$$

$\frac{K_T R}{\beta} = \frac{\beta T}{\rho}$

$R = \frac{\beta^2 \cancel{A} T}{K_T \rho}$

$(c_p - c_v) K_T$

~~5.11~~ 5.13

$$\cancel{d}s = \left.\frac{\partial s}{\partial T}\right|_p dT + \left.\frac{\partial s}{\partial p}\right|_T dp$$

$$T ds = \left.\frac{\partial e}{\partial p}\right|_T dp + \left.\frac{\partial e}{\partial p}\right|_{\cancel{p}} dT - \frac{p}{\rho^2}\left(\frac{\partial \rho}{\partial T} dT + \frac{\partial \rho}{\partial p} dp\right)$$

$$\left.\frac{\partial s}{\partial p}\right|_T = \frac{1}{T}\left.\frac{\partial e}{\partial p}\right|_T - \frac{p}{\rho^{\cancel{2}} T} \cdot \overset{K_T}{\cancel{\frac{\partial \rho}{\partial p}}}\Big|_T$$

5.11 $$\left.\frac{\partial e}{\partial p}\right|_T = K_T\left(\frac{p}{\rho} - \frac{(c_p - c_v)}{\beta}\right) = K_T\left(\frac{p}{\rho} - \frac{\beta^{\cancel{2}} T}{K_T \rho \cancel{\beta}}\right) = \left(\frac{K_T p}{\rho} - \frac{\beta T}{\rho}\right)$$

$$\left.\frac{\partial s}{\partial p}\right|_T = \frac{K_T}{T}\left(\frac{p}{\rho} - \frac{R}{\beta}\right) - \frac{p}{\rho T} K_T = -\frac{K_T (c_p - c_v)}{\beta T}$$

$c_p - c_v = \frac{\beta^2 T}{K_T \rho}$ $$\left.\frac{\partial s}{\partial p}\right|_T = -\frac{\cancel{K_T}}{\beta \cancel{T}}\left(\frac{\beta^{\cancel{2}} \cancel{T}}{\cancel{K_T} \rho}\right) = -\frac{\beta}{\rho}$$

5.13

$$\frac{\partial p}{\partial T}\Big|_\rho = \frac{\beta}{K_T}$$

5.15-5.16

$$ds = \frac{1}{T}\frac{\partial e}{\partial T}\Big|_\rho dT + \frac{1}{T}\left(\frac{\partial e}{\partial \rho}\Big|_T - \frac{P}{\rho^2}\right) d\rho$$

$$\frac{\partial s}{\partial p}\Big|_\rho d\rho + \frac{\partial s}{\partial \rho}\Big|_p dp = \left(\frac{1}{T}\frac{\partial e}{\partial T}\Big|_\rho\right)\left(\frac{\partial T}{\partial \rho}\Big|_p d\rho + \frac{\partial T}{\partial p}\Big|_\rho dp\right) + \frac{1}{T}\left(\frac{\partial e}{\partial \rho}\Big|_T - \frac{P}{\rho^2}\right) d\rho$$

5.15

$$\frac{\partial s}{\partial p}\Big|_\rho = \frac{1}{T}\frac{\partial e}{\partial T}\Big|_\rho \cdot \frac{\partial T}{\partial p}\Big|_\rho$$

$$\frac{\partial s}{\partial \rho}\Big|_p = \frac{1}{T}\frac{\partial e}{\partial T}\Big|_\rho \cdot \frac{\partial T}{\partial \rho}\Big|_p + \frac{1}{T}\left(\frac{\partial e}{\partial \rho}\Big|_T - \frac{P}{\rho^2}\right)$$

$$\frac{\partial e}{\partial T}\Big|_\rho \equiv c_v \qquad \frac{\partial e}{\partial \rho}\Big|_T = \frac{1}{\rho^2}\left(P - \frac{\beta T}{K_T}\right)$$

hence $\frac{\partial s}{\partial p}\Big|_\rho = \frac{1}{T} c_v \left(\frac{K_T}{\beta}\right) = \frac{c_v K_T}{\beta T}$

5.16

$$\frac{\partial s}{\partial \rho}\Big|_p = \frac{1}{T} c_v \left(-\frac{\rho}{\beta}\right) + \frac{1}{T\rho^2}\left(P - \frac{\beta T}{K_T} - P\right)$$

$$= -\frac{c_v \cancel{\rho}}{\rho \beta T} - \frac{\beta \cancel{T}}{\rho^2 K_T \cancel{T}} = -\frac{c_v \rho}{\beta T} - \frac{\beta}{\rho^2 K_T}$$

$$\frac{\beta^2}{\rho K_T} = \frac{1}{\rho}\cdot\frac{\beta}{\rho K_T} = \frac{1}{\rho}\cdot\frac{(c_p - c_v)}{\beta T}$$

$$-\frac{c_v \cancel{\rho}}{\rho \beta T} - \frac{1}{\rho}\cdot\frac{1}{\beta T}(c_p - c_v) = -\frac{c_p}{\rho \beta T}$$

$(4N)^{-1/3} \approx$

$$\left(4 \times 10^{16}\right)^{-1/3}$$

$$= \cancel{\left(\frac{40}{10^{5}}\right)} \quad \left(40 \times 10^{15}\right)^{-1/3}$$

$$= \frac{1}{\left(40 \times 10^{15}\right)^{1/3}} = \frac{10^{-5}}{\sqrt[3]{40}} \qquad \frac{10^{-5}}{3.3}$$

3×3

$4 \times 4 \times 4 = 64$

$3 \times 3 \times 3 = 27 \quad 3.3$

$\ddot{r} - r\dot{\theta}^2 = F/\tilde{m}$

$\ddot{r} - \frac{\dot{\theta}}{r}$

$\frac{1}{2}\dot{r}^2 + \frac{g^2b^2}{2r^2} = \frac{Fr}{\tilde{m}} + c$

$\frac{g^2b^2}{r^2} = \dot{\theta}$

$g^2b^2 = r^4\dot{\theta}^2$

$\frac{\dot{r}^2}{2} + \frac{r^2\dot{\theta}^2}{2}$

$F\dot{r}dt = \int F dr$

$F = -\frac{\partial \phi}{\partial r}$

$\frac{dr}{dt}$

$r^2 d\theta = gb\,dt$

$r^2 \frac{d}{dt} = gb \frac{d}{d\theta}$

$\dot\theta = \frac{gb}{r^2}$

$$\frac{1}{2}\left(\dot r^2 + r^2\dot\theta^2\right) + \frac{C_\alpha}{r^\alpha \tilde m} = \frac{1}{2} g^2$$

$r^2\dot\theta$

$$\frac{1}{2}\left(\dot r^2 + r^2\dot\theta^2\right) = \frac{g^2}{2} - \frac{C_\alpha}{r^\alpha \tilde m}$$

$$r^2\dot\theta^2 = \left(g^2 - \frac{2C_\alpha}{r^\alpha \tilde m}\right) - \dot r^2$$

$$\dot\theta^2 = \left(\frac{g^2}{r^2} - \frac{2C_\alpha}{r^{\alpha+2}\tilde m}\right) - \frac{\dot r^2}{r^2}$$

~~$\frac{gb}{r^2} \cdot \dot\theta =$~~

$$\frac{1}{2}\left(\dot r^2 + \frac{g^2b^2}{r^2}\right) \mp \frac{C_\alpha}{r^\alpha \tilde m} = \frac{1}{2} g^2$$

$$\left(\dot r^{2\,2} + \frac{g^2b^2}{r^2}\right) = g^2 - \frac{2C_\alpha}{r^\alpha \tilde m}$$

$$\dot r^2 = g^2 - \frac{2C_\alpha}{r^\alpha \tilde m} - \frac{g^2b^2}{r^2}$$

$$\dot r = \left(g^2 - \frac{2C_\alpha}{r^\alpha \tilde m} - \frac{g^2b^2}{r^2}\right)^{1/2}$$

$$\dot{\theta}^2 = \frac{g^2}{r^2}\left(1-\frac{2C_\alpha}{g^2 r^\alpha \tilde{m}} - \frac{\dot{r}^2}{g^2}\right)$$

$$\dot{\theta} = \frac{g}{r}$$

$\frac{dr}{d\theta} = 0$ = apse A, $R_0 = b/q_0$ $q = b/r$

$$1 - q_0^2 - (2/\alpha)(q_0/\beta)^\alpha = 0$$

$$\theta_0 = \int_0^{(b/R_0)} \left[1 - q_0^2 - \frac{2}{\alpha}\left(\frac{q_0}{\beta}\right)^\alpha\right]^{-1/2} dq$$

$$\theta = \int_R^\infty \left(1 - \frac{2C_\alpha}{\tilde{m} g^2 r^\alpha} - \frac{b^2}{r^2}\right)^{1/2} \frac{b\,dr}{r^2}$$

$r = b/q$

$dr = -\frac{b}{q^2}dq$

$\cancel{dr = \frac{b}{dq}}$

$dq = -\frac{b\,dr}{r^2}$ $q = \frac{b}{r}$

$$= \int_{r=R}^{\infty}\left(1 - \frac{2C_\alpha}{\tilde{m} g^2 (r/b)^\alpha b^\alpha} - q^2\right)\cancel{\frac{q}{r}\left(-\frac{b}{q^2}dq\right)}(-dq)$$

$r = R$

$q = b/R$

$r = \infty, q = 0$

$$= \int_0^{b/R}\left(1 - \frac{2C_\alpha q^\alpha}{\tilde{m} g^2 b^\alpha} - q^2\right)(+dq) = \int_0^{b/R}\left[1 - q^2 - \frac{2q^\alpha}{\alpha}\left(\frac{\alpha C_\alpha}{\tilde{m} g^2 b^\alpha}\right)\right]$$

$$\frac{1}{\beta^\alpha} = \frac{\alpha C_\alpha}{\tilde{m} g^2 b^\alpha} \qquad \beta^\alpha = b^\alpha\left(\frac{\tilde{m} g^2}{\alpha C_\alpha}\right)$$

$$\sin\frac{1}{2}\chi \cong \frac{\chi}{2} \approx \frac{1}{[1+(1/y^2)]^{1/2}} \approx \frac{1}{(1+x^2)^{1/2}}$$

$$(1+x^2)^{-1/2} \approx \frac{1}{x} \approx y$$

$$x >> 1$$

$$2\pi n_i g t \int_{b_{min}}^{b_{max}} \left(\frac{2Z_1Z_2e^2}{m_e g^2 b}\right)^2 b\,db$$

$$= 8\pi n_i g t \frac{Z_1^2 Z_2^2 e^4}{m_e^2 g^3} \int \frac{b\,db}{b^2}$$

$E = \sum \nu_i \varepsilon_i \qquad \nu_i = \frac{N}{Z} g_i e^{-\beta\varepsilon_i}$

$$E = \frac{N}{Z} \sum g_i \varepsilon_i e^{-\beta\varepsilon_i}$$

$\ln \nu_i! \cong \frac{1}{2} \ln \nu_i + \nu_i \ln \nu_i - \nu_i$

$$\ln W = \sum_i \left\{ \nu_i \ln g_i - \frac{1}{2} \ln \nu_i + \nu_i - \nu_i \ln \nu_i \right\}$$

$$= \sum_i \left\{ \nu_i \ln\left(\frac{g_i}{\nu_i}\right) + \nu_i - \frac{1}{2} \ln \nu_i \right\}$$

$$= N + \sum \left\{ \nu_i \ln\left(\frac{g_i}{\nu_i}\right) - \frac{1}{2} \ln \nu_i \right\}$$

$$= N - \sum \left[\nu_i \ln\left(\frac{\nu_i}{g_i}\right) - \cancel{\ln \nu_i^{1/2}} \right]$$

$$\delta\left[\nu_i \ln\left(\frac{\nu_i}{g_i}\right)\right] = \delta\nu_i \ln\left(\frac{\nu_i}{g_i}\right) + \frac{\nu_i}{\left(\nu_i/g_i\right)} \frac{\delta\nu_i}{g_i}$$

$\ln\left(\frac{\nu_i}{g_i}\right) = \ln\alpha + \beta\varepsilon_0 \qquad = \delta\nu_i \ln\left(\frac{\nu_i}{g_i}\right) + \delta\nu_i$

$\ln\left(\frac{\nu_i}{\alpha g_i}\right) = -\beta\varepsilon_i$

$\frac{\nu_i}{\alpha g_i} = e^{\beta\varepsilon_i}$

$\nu_i = \alpha g_i e^{-\beta\varepsilon_i}$

12.17 $\ln \bar{W} = N - \sum \nu_i \ln\left(\frac{\nu_i}{g_i}\right)$

$$\frac{\nu_i}{g_i} = \frac{N}{Z}\,\bar{e}^{\beta\varepsilon_i}$$

$$\ln\left(\frac{\nu_i}{g_i}\right) = \ln\left(\frac{N}{Z}\right) - \beta\varepsilon_i$$

$$\nu_i \ln\left(\frac{\nu_i}{g_i}\right) = \nu_i \ln\left(\frac{N}{Z}\right) - \beta\nu_i\varepsilon_i$$

$$\sum \nu_i \ln\left(\frac{\nu_i}{g_i}\right) = N\ln\left(\frac{N}{Z}\right) - \beta E$$

$$\ln \bar{W} = N - \left\{N\ln\frac{N}{Z} - \beta E\right\} = N\left(1 + \ln\left(\frac{Z}{N}\right) + \beta E\right)$$

$S = k \ln \bar{W}$

$$\left(\frac{\partial S}{\partial E}\right)_V = \frac{\partial}{\partial E}\left\{Nk\left[1 + \ln Z/N\right]\right\} + \beta k$$

$TdS = dE - pdV$

$$\frac{\partial}{\partial E}\left[Nk\ln\frac{Z}{N}\right] = \frac{Nk}{Z/\cancel{N}}\cdot\frac{1}{\cancel{N}}\frac{\partial Z}{\partial\beta}\cdot\frac{\partial\beta}{\partial E}$$

$\frac{N}{(\cancel{Z}/\cancel{N})}$

~~$Nk\frac{\partial}{\partial E}\left(\ln\frac{N}{Z}\right) = \frac{Nk}{(N/Z)}\left(-\frac{N}{Z^2}\right)\frac{\partial Z}{\partial\beta}\cdot\frac{\partial\beta}{\partial E}$~~

$$Nk\left(\ln Z - \ln N\right) = \frac{Nk}{Z}\frac{\partial Z}{\partial\beta}\cdot\frac{\partial\beta}{\partial E}$$

$$\sum \varepsilon_i g_i e^{-\varepsilon_i/kT} = kT^2 \frac{\partial Z}{\partial T}$$

$$\frac{N}{Z} \sum \varepsilon_i g_i e^{-\varepsilon_i/kT} = \frac{N}{Z} kT^2 \frac{\partial Z}{\partial T}$$

$$\boxed{E} = NkT^2 \frac{\partial \ln Z}{\partial T}$$

$$S = Nk\left[1 + \ln(Z/N)\right] + E/T$$

$$= Nk\left(1 + \ln\frac{Z}{N}\right) + \frac{NkT^2}{T} \frac{\partial \ln Z}{\partial T}$$

$$= Nk\left\{1 + \ln\frac{Z}{N} + T \frac{\partial \ln Z}{\partial T}\right\}$$

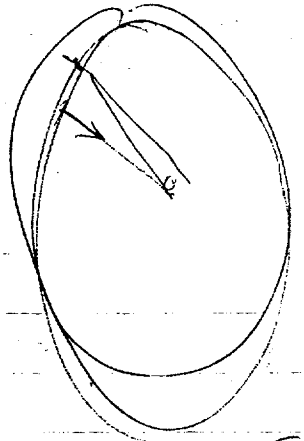
$S =$

$$TdS = dE + pdV$$

$$T\left(\frac{\partial S}{\partial V} dV + \frac{\partial S}{\partial T} dT\right) = \left(\frac{\partial E}{\partial V} dV + \frac{\partial E}{\partial T} dT\right) + pdV$$

$$pdV = T\frac{\partial S}{\partial V} dV - \frac{\partial E}{\partial V} dV$$

$$T\frac{\partial S}{\partial V} = T\,Nk \frac{\partial \ln(Z/N)}{\partial V} + NkT^2 \frac{\partial^2 \ln Z}{\partial T \partial V}$$



$$\frac{m_{(1)}^2}{e^4 Z_1^2 Z_2^2 n_e} = \frac{1}{m_1^2 \cdot N G^2}$$

$$\frac{1}{8\pi n w p_0^2} = \frac{1}{8\pi n w} \cdot \frac{(m w^2)^2}{(Z_1 Z_2 e^2)^2}$$

$$= \frac{m^2 w^3}{8\pi Z_1^2 Z_2^2 e^4\, n}$$

$$\frac{Z_1 Z_2 e^2}{m w^2}$$

$$b_{max} = D = \left[\frac{kT}{4\pi e^2 (2n_e)}\right]^{1/2}$$

$$b_{min} = \frac{Z_i e^2}{m_e g^2}$$

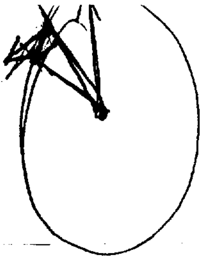
$$\Lambda = \frac{b_{max}}{b_{min}} = \left[\frac{kT}{8\pi e^2 n_e}\right]^{1/2} \cdot \frac{m_e g^2}{e^2}$$

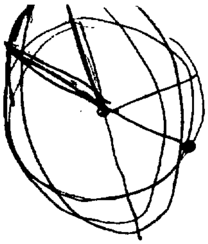
$$g^2 = \frac{3kT}{m_e} =$$

$$\Lambda = \left[\frac{kT}{8\pi \cancel{e}^2 m_e}\right]^{1/2} \frac{\cancel{m_e}\,(3kT/\cancel{m_e})}{e^2}$$

$$= \left[\frac{kT}{8\pi m_e}\right]^{1/2} \frac{1}{e} \frac{(3kT)}{e^2}$$

$$= \frac{3}{e^3}\left[\frac{k^3 T^3}{8\pi m_e}\right]^{1/2}$$





$v = r \cdot \theta$

$dx = r\,d\theta \cdot dr$

p. 22 Lagrangean? spelling?

p. 22-23 introduce Liouville eqn + BBGKY hierarchy. Are you going to develop these or drop them. Need statement of intent.

$$m_1 m_2 \ddot{\underset{\sim}{x}} = (m_1 + m_2) \underset{\sim}{F}$$

$$x = r$$

$$\sum x^2 = \frac{8\pi e^4 n_i t}{m_e^2 g^3} \ln\Lambda$$

$$\frac{\partial x^2}{\partial t} = \frac{8\pi e^4 n_i}{m_e^2 g^3} \ln\Lambda$$

$$t_D = \frac{m_e^2 g^3}{8\pi e^4 n_i \ln\Lambda}$$

$$\frac{e^2}{m_e g^2} \qquad t_s = \frac{1}{\pi b^2 n g} = \frac{1}{\pi n_p g \left(\frac{e^2}{[illegible]}\right)^2}$$

$$= \frac{m_e^2 g^3}{\pi n_p e^4}$$

$$b_{max} = D$$

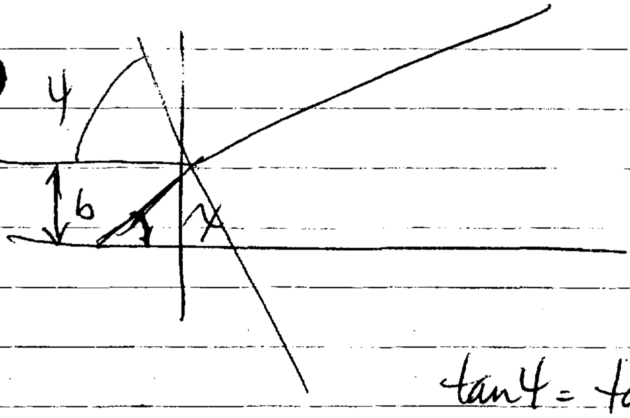
$$b_{min} = \frac{Z_i e^2}{m_e g^2}$$

$$\tan\frac{\chi}{2} = \frac{Z_1 Z_2 e^2}{\tilde{m} g^2 b} \qquad \Lambda = \frac{b_{max}}{b_{min}} \cong \frac{4.8(T/n_e)^{1/2}}{Z_i e^2/m_e g^2}$$

$$\chi = \qquad\qquad = \frac{4.8\, m_e g^2 (T/n_e)^{1/2}}{Z_i e^2}$$

$$\sin\frac{\chi}{2} = \left[1 + \left(\frac{\tilde{m} g^2 b}{Z_1 Z_2 e^2}\right)^2\right]^{1/2} \qquad = \frac{4.8\, m_e g^2}{Z_i e^2}\left(\frac{T}{n_e}\right)^{1/2}$$

$$\sin\chi = 2\sin\frac{\chi}{2}\cos\frac{\chi}{2} \qquad n_e + n_i = 2n_e \qquad D = \left[\frac{kT}{8\pi e^2 n_e}\right]^{1/2}$$



$$\psi = \frac{\pi - \chi}{2} = \frac{\pi}{2} - \frac{\chi}{2}$$

$$\tan\psi = \tan\left(\frac{\pi}{2} - \frac{\chi}{2}\right) = \frac{\sin(\pi/2 - \chi/2)}{\cos(\pi/2 - \chi/2)}$$

$$= \frac{\sin\pi/2 \cos(-\chi/2) + \cos\pi/2 \sin(-\chi/2)}{\cos\pi/2 \cos(-\chi/2) - \sin(\pi/2)\sin(-\chi/2)}$$

$$= \frac{\cos(-\chi/2)}{-\sin(-\chi/2)} = \frac{\cos\chi/2}{\sin\chi/2} = \cot\chi/2$$

$$\frac{(1-\sin^2\chi/2)^{1/2}}{\sin\chi/2} = x$$

$$(1-\sin^2\chi/2)^{1/2} = x\sin\chi/2$$

$$1-\sin^2\chi/2 = x^2\sin^2\chi/2$$

$\Pi_e \cong (1+\psi)$

$\Pi_i \cong 1 - Z_i\psi$

$\rho = -e n_e \Pi_e + e \sum n_i Z_i \Pi_i$

$\cong -e n_e (1+\psi) + e \sum n_i Z_i (1 - Z_i\psi)$

$= -e n_e - e n_e \psi + e \sum n_i Z_i - e \sum n_i Z_i^2 \psi$

$= -e n_e \psi - e \sum n_i Z_i^2 \psi$

$\psi = \frac{Ze\phi}{kT}$ $\qquad \phi = \frac{kT\psi}{Ze}$

$4\pi\rho = -\frac{\phi}{D^2} = \boxed{\frac{kT\cancel{\psi}}{Ze D^2}} = 4\pi\left[-e\cancel{\psi}\left(n_e + \sum Z_i^2 n_i\right)\right]$

$\frac{kT}{Ze} = 4\pi e (n_e + \sum Z_i^2 n_i) D^2$

$D^2 = \frac{kT}{Ze \cdot 4\pi e (n_e + \sum Z_i^2 n_i)}$

$D^2 = \frac{kT}{4\pi Z e^2 (n_e + \sum Z_i^2 n_i)}$

$t_D = 1 / \Sigma \chi^2$

$w^2 = \frac{Z Z_1 e^2}{m b}$

$\Sigma (\Delta E)^2$

$\langle (\Delta w_T)^2 \rangle = 8\pi n_1 w^3 b^2 \ln \Lambda$

$\Sigma \chi^2 = 2\pi n_2 g t \int_{b_{min}}^{b_{max}} \chi^2(g, b)\, b\, db$

$w^2 = \frac{Z^2 e^2}{m b}$

$= (8\pi Z_i^2 e^4 n_i t / m_e^2 g^3) \ln \Lambda$

$\langle (\Delta w_T)^2 \rangle = \frac{8\pi n_1 \cancel{wb} Z^2 e^2}{m}$

$= \frac{8\pi n Z^2 e^2 (wb)}{m} \ln \cancel{\Lambda}$

$1 / (8\pi Z_i^2 e^4 n_i t / m_e^2 g^3) \ln \Lambda$

$wb \overset{?}{=} \frac{e^2}{m_e g^3}$

$t_D = m_e^2 g^3 / (8\pi Z_i^2 e^4 n_i \boxed{t} \ln \Lambda)$

$\Sigma \chi^2 (t_D) \approx 1$

$\Sigma (\Delta E)^2 (t_E) = E^2$

$t_D =$

$1 / \left[\frac{d}{dt} \Sigma \chi^2 \right]$

$t_E = E^2 / \left[\frac{d}{dt} \Sigma \Delta E^2 \right]$

$\boxed{4144-4160}$

$$\frac{\lambda^3 N}{V} = \left(\frac{2\pi\hbar^2}{mkT}\right)^{3/2}\frac{N}{V} = z$$

$$P = T\frac{\partial S}{\partial V} - \frac{\partial E}{\partial V} \qquad Q_N^{(1/N)} = \frac{V}{N}\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2}$$

$$\int_0^\infty e^{-mu^2/2kT} u^2 du$$

$$S = Nk\left(1 + \ln Z/N\right) + E/T$$

$$d^6N = g_i e^{-\beta\varepsilon_i}\frac{N}{Z}$$

$$\frac{\partial S}{\partial V} = Nk\frac{\partial \ln(Z/N)}{\partial V} + \frac{\cancel{1}}{T}\frac{\partial E}{\partial V}$$

$$\frac{d^6N}{N} = \frac{g_i e^{-\beta\varepsilon_i}}{Z}$$

$$Z = \sum g_i e^{-\beta\varepsilon_i}$$

$$P = NkT\frac{\partial \ln(Z/N)}{\partial V} + \frac{\partial E}{\partial V} - \frac{\partial E}{\partial V}$$

$$g_i = \left(\frac{m}{h}\right)^3 d^3x\, d^3u = \left(\frac{4\pi m^3}{h^3}\right) d^3x\, u^2du$$

$$\frac{1}{kT}$$

$$\frac{\partial\beta}{\partial T} = -\frac{1}{kT^2}$$

$$S = Nk\ln\left[\frac{V}{N}\left(\frac{3}{2}kT\right)^{3/2}\right] + Nk\left(\frac{5}{2}\right) + \frac{3}{2}Nk\ln\left(\frac{4\pi m}{3h^2}\right)$$

$$\cancel{\beta = \frac{1}{kT}} \qquad T = \frac{1}{\beta k}$$

$$= \frac{5}{2}Nk + Nk\ln\frac{V}{N} + Nk\left[\ln\left\{\left(\frac{3}{2}kT\right)^{3/2}\right\} + \ln\left(\frac{4\pi m}{3h^2}\right)^{3/2}\right]$$

$$= \frac{5}{2}Nk + Nk\ln\frac{V}{N} + Nk\ln\left(\frac{2\pi mkT}{h^2}\right)^{3/2}$$

$$E = -\frac{\partial \ln Z}{\partial\beta}$$

$$= -\frac{\partial \ln Z}{\partial T}\cdot\frac{\partial T}{\partial\beta}$$

$$+ kT^2\frac{\partial \ln Z}{\partial T}$$

$$d^6N = f(u)\, d^3x\, d^3u$$

$$Z_{tr} = \left(\frac{2\pi m kT}{h^2}\right)^{3/2} V$$

$$\underline{d^6N} = f(u)\, dV\, d^3u = \frac{\cancel{4\pi} m^3 N}{h^3} \frac{1}{\left(\frac{2\pi m kT}{h^2}\right)^{3/2} V} e^{-mu^2/2kT}\, dV\, \cancel{u^2}\, d^3u$$

$$= \frac{m^3 N}{\cancel{h^3} V} \left(\frac{\cancel{h}^2}{2\pi m kT}\right)^{3/2} e^{-mu^2/2kT}\, dV\, d^3u$$

$$f(u)\, dV\, d^3u = \frac{N}{V}\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mu^2/2kT}\, dV\, d^3u$$

$$\frac{S}{Nm} = \frac{\cancel{N}k}{\cancel{N}m} \left\{\frac{5}{2}\cancel{*} - \ln N + \frac{3}{2}\ln T + \frac{3}{2}\ln\left(\frac{2\pi m \cancel{k}}{h^2}\right) + \frac{3}{2}\ln k\right\}$$

$$= R\left\{\frac{5}{2} - \ln N + \ln(NkT) - \ln(NkT) + \frac{3}{2}\ln T + \frac{3}{2}\ln\left(\frac{2\pi m}{h^2}\right) + \frac{3}{2}\ln k\right\}$$

$$= R\left\{\overset{\checkmark}{\frac{5}{2}} - \cancel{\ln N} + \cancel{\ln N} \overset{\checkmark}{+ \ln k} \overset{\checkmark}{+ \ln T} \overset{\checkmark}{- \ln p} + \frac{3}{2}\overset{\checkmark}{\ln T} + \frac{3}{2}\ln\left(\frac{2\pi m}{h^2}\right) \overset{\checkmark}{+ \frac{3}{2}\ln k}\right\}$$

$$= R\left\{\frac{5}{2} + \frac{5}{2}\ln T + \frac{5}{2}\ln k - \ln p + \frac{3}{2}\ln\left(\frac{2\pi m}{h^2}\right)\right\}$$

$$g_i = \frac{\cancel{4\pi} m^3}{h^3}\, d^3x\, d^3u$$

$$S = k\ln\left\{\textcircled{N} - \sum \nu_i \ln(\nu_i / g_i)\right\}$$

$$\sum(\nu_i^0 + \Delta\nu_i)\left[\ln\nu_i^0 + \frac{\Delta\nu_i}{\nu^0} - \frac{1}{2}\left(\frac{\Delta\nu}{\nu_0}\right)^2\right]$$

$$= \Big\{\sum\nu^0\ln\nu^0 + \sum\Delta\nu\ln\nu + \sum\nu\frac{\Delta\nu}{\nu} + \sum\frac{\Delta\nu^2}{\nu}$$

$$- \frac{1}{2}\sum\frac{\Delta\nu^2}{\nu} - \frac{1}{2}\Delta\nu\left(\frac{\Delta\nu}{\nu}\right)^2\Big\}$$

$$= \sum\nu\ln\nu + \sum\Delta\nu\ln\nu + \sum\Delta\nu + \frac{1}{2}\sum\frac{\Delta\nu^2}{\nu}$$

$$\underbrace{N - \sum\nu\ln\nu}_{\ln W_0} - \underbrace{\sum\Delta\nu\ln\nu}_{\substack{\parallel\\0}} - \underbrace{\sum\Delta\nu}_{\substack{\parallel\\0}} - \frac{1}{2}\sum\frac{\Delta\nu^2}{\nu}$$

$$\ln W \cong \ln W_0 - \frac{1}{2}\sum\frac{\Delta\nu^2}{\nu}$$

$$\ln\left(\frac{N}{W_0}\right) =$$

$$E = \frac{N}{Z}\sum\varepsilon_i g_i e^{-\varepsilon_i/kT}$$

$$Z = \sum g_i e^{-\varepsilon_i/kT}$$

$$\frac{\partial Z}{\partial T} = -\sum\frac{\varepsilon_i(-1)}{kT^2}g_i e^{-\varepsilon_i/kT}$$

$$= +\frac{1}{kT^2}\sum\varepsilon_i g_i e^{-\varepsilon_i kT}$$

Chapter 3

$v_1 = v_r$
$v_2 = r v_\theta$
$v_3 = r \sin\theta \, v_\phi$

25.9

$$E_{ij} = \frac{1}{2}(v_{i;j} + v_{j;i})$$

$$E_{11} = v_{1;1} = \frac{\partial v_r}{\partial r}$$

$$E_{21} = E_{12} = \frac{1}{2}\left(v_{1;2} + v_{2;1}\right) = \frac{1}{2}\left(v_{1,2} + v_{2,1}\right) - \frac{1}{2}\left[\left\{ {k \atop 12} \right\} v_k + \left\{ {k \atop 21} \right\} v_k\right]$$

$$= \frac{1}{2}\left[\frac{\partial v_1}{\partial \theta} + \frac{\partial v_2}{\partial r}\right] - \frac{1}{2}\left[2\left\{ {2 \atop 12} \right\} v_2\right] = \frac{1}{2}\left[\frac{\partial v_1}{\partial \theta} + \frac{\partial v_2}{\partial r}\right] - \frac{v_2}{r}$$

$$E_{31} = E_{13} = \frac{1}{2}\left[\frac{\partial v_1}{\partial \phi} + \frac{\partial v_3}{\partial r}\right] - \frac{1}{2}\left[\left\{ {k \atop 13} \right\} v_k + \left\{ {k \atop 31} \right\} v_k\right]$$

$$= \frac{1}{2}\left[\frac{\partial v_1}{\partial \phi} + \frac{\partial v_3}{\partial r}\right] - \frac{v_3}{r}$$

$$E_{22} = \frac{\partial v_2}{\partial \theta} - \left\{ {k \atop 22} \right\} v_k = \frac{\partial v_2}{\partial \theta} + r v$$ [illegible]

$$E_{23} = \frac{1}{2}\left[\frac{\partial v_2}{\partial \phi} + \frac{\partial v_3}{\partial \theta}\right] - \left\{ {k \atop 23} \right\} v_k = \frac{1}{2}\left[\frac{\partial v_2}{\partial \phi} + \frac{\partial v_3}{\partial \theta}\right] - \cot\theta \, v_3$$

$$E_{33} = \frac{\partial v_3}{\partial \phi} - \frac{1}{2}\left[\left\{ {k \atop 33} \right\} v_k\right] = \frac{\partial v_3}{\partial \phi} - \left\{ {1 \atop 33} \right\} v_1 - \left\{ {2 \atop 33} \right\} v_2$$

$$= \frac{\partial v_3}{\partial \phi} + r \sin^2\theta \, v_1 + \sin\theta\cos\theta \, v_2$$

$E_{11} = E_{rr}$ $E_{21} = E_{12} = E_{r\theta} \, r$ $E_{13} = E_{31} = E_{r\phi} \cdot r\sin\theta$

$E_{22} = E_{\theta\theta} r^2$ $E_{23} = E_{32} = E_{\theta\phi} r^3 \sin\theta$ $E_{33} = E_{\phi\phi} r^3 \sin^2\theta$

25.10

Thus

$$E_{rr} = \partial v_r / \partial r$$

$$rE_{r\theta} = \frac{1}{2}\left[\frac{\partial v_r}{\partial \theta} + \frac{\partial}{\partial r}(rv_\theta)\right] - \frac{rv_\theta}{r}$$

$$\Rightarrow E_{r\theta} = E_{\theta r} = \frac{1}{2}\left[\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} - \frac{2 v_\theta}{r}\right] = \frac{1}{2}\left[\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right]$$

$$r\sin\theta\, E_{r\phi} = \frac{1}{2}\left[\frac{\partial v_r}{\partial \phi} + \frac{\partial}{\partial r}(r\sin\theta v_\phi)\right] - \frac{r\sin\theta v_\phi}{r}$$

$$\Rightarrow \quad E_{r\phi} = \frac{1}{2r\sin\theta}\left[\frac{\partial v_r}{\partial \phi} + \sin\theta . v_\phi + r\sin\theta\frac{\partial v_\phi}{\partial r} - 2\sin\theta v_\phi\right]$$

$$E_{\phi r} = E_{r\phi} = \frac{1}{2}\left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{\partial v_\phi}{\partial r} - \frac{v_\phi}{r}\right]$$

$$r^2 E_{\theta\theta} = \frac{\partial (rv_\theta)}{\partial \theta} + rv_r = r\frac{\partial v_\theta}{\partial \theta} + rv_r$$

$$\Rightarrow E_{\theta\theta} = \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}$$

$$E_{\theta\phi}\, r^2\sin\theta = \frac{1}{2}\left[\frac{\partial(rv_\theta)}{\partial\phi} + \frac{\partial(r\sin\theta v_\phi)}{\partial\theta}\right] - \cot\theta . r\sin\theta v_\phi$$

$$= \frac{1}{2}\left[r\frac{\partial v_\theta}{\partial\phi} + r\cos\theta v_\phi + r\sin\theta\frac{\partial v_\phi}{\partial\theta} - 2r\overset{\cos\theta}{\cancel{\cot\theta\sin\theta}}v_\phi\right]$$

$$\Rightarrow E_{\phi\theta} = E_{\theta\phi} =$$

$$= \frac{1}{2}\left[\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi} + \frac{1}{r}\frac{\partial v_\phi}{\partial\theta} - \frac{1}{r}\cot\theta v_\phi\right]$$

$$r^2\sin^2\theta\, E_{\phi\phi} = \frac{\partial(r\sin\theta\, v_\phi)}{\partial\phi} + r\sin^2\theta\, v_r + \sin\theta\cos\theta \cdot r\, v_\theta$$

$$\Rightarrow E_{\phi\phi} = \frac{r\sin\theta}{r^2\sin^2\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_r}{r} + \frac{\cot\theta}{r} v_\theta$$

$$E_{\phi\phi} = \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_r}{r} + \frac{\cot\theta}{r} v_\theta$$

end 25.10

25.11

$$T_{ij} = -p g_{ij} + \sigma_{ij}$$

$$= -p g_{ij} + 2\mu E_{ij} + \left(\zeta - \tfrac{2}{3}\mu\right) v^k_{;k}\, g_{ij}$$

$$\left[\begin{array}{lll} T_{11} = T_{rr} & T_{12} = T_{21} = r T_{r\theta} & T_{13} = T_{31} = r\sin\theta\, T_{r\phi} \\ T_{22} = r^2 T_{\theta\theta} & T_{23} = T_{32} = r^2\sin\theta\, T_{\theta\phi} & T_{33} = r^2\sin^2\theta\, E_{\phi\phi} \end{array}\right.$$

$g_{11} = 1 \quad g_{22} = r^2 \quad g_{33} = r^2\sin^2\theta \quad g_{ij} = 0 \quad i \neq j.$

$$T_{11} = -p g_{11} + 2\mu E_{11} + \left(\zeta - \tfrac{2}{3}\mu\right)\overset{\nabla\cdot \underset{\sim}{v}}{\boxed{v^k_{;k}}}\, g_{11}$$

$$T_{rr} = -p + 2\mu\frac{\partial v_r}{\partial r} + \left(\zeta - \tfrac{2}{3}\mu\right)\cdot\nabla\cdot\underset{\sim}{v}$$

$$r^2 T_{22} = \cancel{r^2} T_{\theta\theta} = -p\cancel{r^2} + 2\mu\, \cancel{r^2} E_{\theta\theta} + \left(\zeta - \tfrac{2}{3}\mu\right)\nabla\cdot\underset{\sim}{v}\, \cancel{r^2}$$

$$T_{\theta\theta} = -p + 2\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r}{r}\right) + \left(\zeta - \tfrac{2}{3}\mu\right)\nabla\cdot\underset{\sim}{v}$$

$$T_{33} = r^2\sin^2\theta\, T_{\phi\phi} = -p\, r^2\sin^2\theta + 2\mu\, r^2\sin^2\theta\, E_{\phi\phi} + \left(\zeta - \tfrac{2}{3}\mu\right)\nabla\cdot\underset{\sim}{v}\cdot r^2\sin^2\theta$$

$$T_{\phi\phi} = -p + 2\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_r}{r} + \frac{\cot\theta\, v_\theta}{r}\right) + \left(\zeta - \tfrac{2}{3}\mu\right)\nabla\cdot\underset{\sim}{v}$$

$$T_{12}=T_{21}=rT_{r\theta}=2\mu rE_{r\theta}=2\mu r\cdot\frac{1}{2}\left[\frac{1}{r}\frac{\partial v_r}{\partial\theta}+\frac{\partial v_\theta}{\partial r}-\frac{v_\theta}{r}\right]$$

$$T_{r\theta}=\mu\left[\frac{1}{r}\frac{\partial v_r}{\partial\theta}+\frac{\partial v_\theta}{\partial r}-\frac{v_\theta}{r}\right]=\cancel{\frac{\mu}{r}\left[\frac{\partial v_r}{\partial\theta}+\frac{\partial}{\partial r}(v_\theta)\right]}$$

$$=\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{1}{r}\frac{\partial v_r}{\partial\theta}\right]$$

$$T_{13}=T_{31}=r\sin\theta\, T_{r\phi}=2\mu\cdot r\sin\theta\, E_{r\phi}=2\mu\frac{r\sin\theta}{2}\left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\theta}+r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]$$

$$T_{r\phi}=\mu\left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi}+r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]$$

$$T_{23}=T_{32}=r^2\sin\theta\, T_{\theta\phi}=2\mu\cdot r^2\sin\theta\, E_{\theta\phi}$$

$$T_{\theta\phi}=2\mu\cdot\frac{1}{2}\left[\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}+\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right)\right]$$

$$=\mu\left[\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}+\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right)\right]$$

Chapter 3

$$\rho \frac{Dv^i}{Dt} = f^i + T^{ji}{}_{,j}$$

1-D planar

$$\rho \frac{Dv_z}{Dt} = f_z - \frac{\partial p}{\partial z} + \frac{\partial}{\partial z}\left[2\mu \frac{\partial v_z}{\partial z}\right] + \frac{\partial}{\partial z}\left(\zeta - \frac{2}{3}\mu\right)\frac{\partial v_z}{\partial z}$$

$$= f_z - \frac{\partial p}{\partial z} + \frac{\partial}{\partial z}\left\{2\mu \frac{\partial v_z}{\partial z} + \left(\zeta - \frac{2}{3}\mu\right)\frac{\partial v_z}{\partial z}\right\}$$

26.2 $$= f_z - \frac{\partial p}{\partial z} + \frac{\partial}{\partial z}\left\{\left(\zeta + \frac{4}{3}\mu\right)\frac{\partial v_z}{\partial z}\right\}$$

μ + ζ constant

$$T_{ij,j} = \mu\left(v_{i,j} + v_{j,i} - \frac{2}{3} v_{k,k}\delta_{ij}\right)_{,j} + \zeta v_{k,ki}$$

$$= \mu v_{i,jj} + \underbrace{\mu v_{j,ij}}_{\mu v_{k,ki}} - \frac{2}{3}\mu v_{k,ki} + \zeta v_{k,ki}$$

$$= \mu v_{i,jj} + \frac{1}{3}\mu v_{k,ki} + \zeta v_{k,ki}$$

$$= \mu v_{i,jj} + \left(\zeta + \frac{1}{3}\mu\right) v_{k,ki}$$

26.5 $$= \mu \nabla^2 v_i + \left(\zeta + \frac{1}{3}\mu\right)(\nabla \cdot v)_{,i}$$

So
Constant viscosity coeffs $\Rightarrow$

$$\rho \frac{Dv}{Dt} = f - \nabla p + \mu \nabla^2 v + \left(\zeta + \frac{1}{3}\mu\right)\nabla(\nabla \cdot v)$$

A3.91 $$T^{1j}{}_{;j} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 T_{rr}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, T_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial T_{r\phi}}{\partial\phi} - \frac{1}{r}\left(T_{\theta\theta} + T_{\phi\phi}\right)$$

$$T^{2j}{}_{;j} = \frac{1}{r}\left[\frac{1}{r^2}\frac{\partial(r^2 T_{r\theta})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, T_{\theta\theta}) + \frac{1}{r\sin\theta}\frac{\partial T_{\theta\phi}}{\partial\phi} + \frac{1}{r}\left(T_{r\theta} - \cot\theta\, T_{\phi\phi}\right)\right]$$

$$T^{3j}{}_{;j} = \frac{1}{r\sin\theta}\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 T_{r\phi}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, T_{\theta\phi}) + \frac{1}{r\sin\theta}\frac{\partial T_{\phi\phi}}{\partial\phi} + \frac{1}{r}\left(T_{r\phi} + \cot\theta\, T_{\theta\phi}\right)\right]$$

$$\rho a^i = f^i + T^{ji}{}_{;j}$$

Use 25.10 and A3.91:

$$T^{1j}{}_{;j} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 T_{rr}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, T_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial T_{r\phi}}{\partial\phi} - \frac{1}{r}(T_{\theta\theta} + T_{\phi\phi})$$

$\eta \equiv (\zeta - \frac{2}{3}\mu)\nabla\cdot v$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(-p + 2\mu\frac{\partial v_r}{\partial r} + \eta\right)\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\,\mu\left(r\frac{\partial}{\partial r}(v_\theta/r) + \frac{1}{r}\frac{\partial v_r}{\partial\theta}\right)\right]$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\mu\left(r\frac{\partial}{\partial r}(v_\phi/r) + \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right)\right] - \frac{1}{r}\left[-p + 2\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r}{r}\right) + \eta\right.$$

$$\left. - p + 2\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_r}{r} + \frac{v_\theta\cot\theta}{r}\right) + \eta\right]$$

$$= \cancel{-\frac{2p}{r}} - \frac{\partial p}{\partial r} + \cancel{\frac{2p}{r}} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(2\mu r^2\frac{\partial v_r}{\partial r}\right) - \frac{2}{r}\cdot\frac{2\mu v_r}{r} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\eta) - \frac{2\eta}{r}$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\mu\sin\theta}{r}\frac{\partial v_r}{\partial\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left(\frac{\mu}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right)$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\mu\sin\theta\, r\frac{\partial}{\partial r}(v_\theta/r)\right) \boxed{- \frac{2\mu}{r^2}v_\theta\cot\theta - \frac{2\mu}{r^2}\frac{\partial v_\theta}{\partial\theta}}$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left(\mu r\frac{\partial}{\partial r}(v_\phi/r)\right) - \frac{2\mu}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(2\mu r^2\frac{\partial v_r}{\partial r}\right) - \frac{4\mu v_r}{r^2} = \frac{2}{r^2}\left[\frac{\partial}{\partial r}\left(\mu r^2\frac{\partial v_r}{\partial r}\right) - 2\mu v_r\right]$$

$$= \frac{2}{r^2}\left[\frac{\partial}{\partial r}\left(\mu r^2\frac{\partial v_r}{\partial r}\right) - \frac{\mu v_r}{r}\frac{\partial r^2}{\partial r}\right]$$

$$= 2\frac{\partial}{\partial r}\left(\mu\frac{\partial v_r}{\partial r}\right) + \frac{4}{r}\mu\frac{\partial v_r}{\partial r} - \frac{4\mu v_r}{r^2} = \frac{\partial}{\partial r}\left(2\mu\frac{\partial v_r}{\partial r}\right) + \frac{4\mu}{r}\left(\frac{\partial v_r}{\partial r} - \frac{v_r}{r}\right)$$

✓ [illegible] $$= \frac{\partial}{\partial r}\left(2\mu\frac{\partial v_r}{\partial r}\right) + 4\mu\,\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)$$

r $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(2\mu r^2\frac{\partial v_r}{\partial r}\right)-\frac{4\mu v_r}{r^2}=\frac{\partial}{\partial r}\left(2\mu\frac{\partial v_r}{\partial r}\right)+4\mu\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)$$

η $$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\eta)-\frac{2\eta}{r}=\frac{2\eta}{r}+\frac{\partial\eta}{\partial r}-\frac{2\eta}{r}=\frac{\partial\eta}{\partial r}$$

θ $$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\mu\sin\theta}{r}\frac{\partial v_r}{\partial\theta}\right)=\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{\mu}{r}\frac{\partial v_r}{\partial\theta}\right)+\frac{\mu\cot\theta}{r^2}\frac{\partial v_r}{\partial\theta}$$

leave alone (φ) $$\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left(\frac{\mu}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\mu r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]-\frac{2\mu}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

θ $$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\mu r\sin\theta\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]=\frac{1}{r}\frac{\partial}{\partial\theta}\left[\mu r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]+\mu\cot\theta\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)$$

θ $$-\frac{2\mu}{r^2}\cot\theta\, v_\theta-\frac{2\mu}{r^2}\frac{\partial v_\theta}{\partial\theta}=-\frac{2\mu}{r^2}\left(\cot\theta\, v_\theta+\frac{\partial v_\theta}{\partial\theta}\right)=-\frac{2\mu}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, v_\theta\right)$$

$$T^{1j}{}_{;j}=-\frac{\partial p}{\partial r}+\frac{\partial}{\partial r}\left(2\mu\frac{\partial v_r}{\partial r}+(\zeta-\tfrac{2}{3}\mu)\nabla\cdot\underset{\sim}{v}\right)+\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{\mu}{r}\frac{\partial v_r}{\partial\theta}+\mu r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]$$

$$+\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\frac{\mu}{r\sin\theta}\frac{\partial v_r}{\partial\phi}+\mu r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]$$

$$+\frac{\mu\cot\theta}{r^2}\frac{\partial v_r}{\partial\theta}+4\mu\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)-\frac{2\mu}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}+\mu\cot\theta\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)-\frac{2\mu}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, v_\theta\right)$$

$$T^{1j}{}_{;j}=-\frac{\partial p}{\partial r}+\frac{\partial}{\partial r}\left[2\mu\frac{\partial v_r}{\partial r}+(\zeta-\tfrac{2}{3}\mu)\underset{\sim}{\nabla}\cdot\underset{\sim}{v}\right]+\frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{\mu}{r}\frac{\partial v_r}{\partial\theta}+\mu r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]+\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\frac{\mu}{\sin\theta}\frac{\partial v_r}{\partial\phi}+\mu r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]$$

$$+\frac{\mu}{r}\left[4r\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)-\frac{2}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, v_\theta\right)-\frac{2\cancel{\mu}}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}+\frac{\cot\theta}{r}\frac{\partial v_r}{\partial\theta}+r\cot\theta\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]$$

$$rT^{2j}{}_{;j}=\cancel{r}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2T_{r\theta}\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, T_{\theta\theta}\right)+\frac{1}{r\sin\theta}\frac{\partial T_{\theta\phi}}{\partial\phi}+\frac{1}{r}\left(T_{r\theta}-\cot\theta\, T_{\phi\phi}\right)\right.$$

$$=\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{1}{r}\frac{\partial v_r}{\partial\theta}\right)\right]+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\left(-p+\frac{2\mu}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{2\mu v_r}{r}\right)+\eta\right]$$

$$-\frac{\cot\theta}{r}\left[-p+\frac{2\mu}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}+\frac{2\mu v_r}{r}+\frac{2\mu\cot\theta}{r}v_\theta+\eta\right]$$

$$+\frac{1}{r}\left[\mu r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{\mu}{r}\frac{\partial v_r}{\partial\theta}\right]+\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\frac{\mu\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right)+\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right]$$

$$T^{2j}{}_{;j} = \frac{\partial}{\partial r}\left[\mu r \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right] + \frac{2\mu}{r} r \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{\partial}{\partial r}\left(\frac{\mu}{r}\frac{\partial v_r}{\partial \theta}\right) + \frac{2\mu}{r}\cdot\frac{1}{r}\frac{\partial v_r}{\partial \theta}$$

$$+ \frac{1}{r}\frac{\partial}{\partial \theta}\left[-p + \frac{2\mu}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{2\mu v_r}{r} + \eta\right] + \frac{\cot\theta}{r}\left[\cancel{-p} + \frac{2\mu}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{2\mu v_r}{r} \cancel{+\eta}\right]$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left[\frac{\mu \sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]$$

$$+ \frac{\mu}{r}\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] + \cancel{\frac{p\cot\theta}{r}} - \frac{\mu}{r}\left[\frac{2\cot\theta}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{2\cot\theta}{r} v_r + \frac{2\cot^2\theta}{r} v_\theta\right] - \cancel{\frac{\eta\cot\theta}{r}}$$

$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{\partial}{\partial r}\left[\mu r \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{\mu}{r}\frac{\partial v_r}{\partial \theta}\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[\frac{2\mu}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{2\mu v_r}{r} + \eta\right]$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left[\frac{\mu\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{\mu}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]$$

$$+ \frac{\mu}{r}\left[\frac{2}{r}\frac{\partial v_r}{\partial \theta} + 2r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{2\cot\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \cancel{\frac{2\cot\theta}{r}} v_r + r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta} - \frac{2\cot\theta}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right.$$

$$\left. - \cancel{\frac{2\cot\theta}{r}} v_r - \frac{2\cot^2\theta}{r} v_\theta\right]$$

$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{\partial}{\partial r}\left[\mu r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{\mu}{r}\frac{\partial v_r}{\partial \theta}\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[\frac{2\mu}{r}\left(\frac{\partial v_\theta}{\partial \theta} + v_r\right) + \left(\zeta - \frac{2}{3}\mu\right)\nabla\cdot\underline{v}\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left[\frac{\mu\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{\mu}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]$$

$$+ \frac{\mu}{r}\left[\frac{3}{r}\frac{\partial v_r}{\partial \theta} + \cancel{\frac{2\cot\theta}{r}} v_r + 3r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{2\cot\theta}{r}\frac{\partial v_\theta}{\partial \theta} - \frac{2\cot\theta}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} - \frac{2\cot^2\theta}{r} v_\theta\right]$$

$$= \frac{\mu}{r}\left[\frac{3}{r}\frac{\partial v_r}{\partial \theta} + 3r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{2\cot\theta}{r}\left(\frac{\partial v_\theta}{\partial \theta} - \cot\theta\, v_\theta - \frac{1}{\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right)\right]$$

$$= \frac{\mu}{r}\left[\frac{3}{r}\frac{\partial v_r}{\partial \theta} + 3r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{2\cot\theta}{r}\left(\sin\theta\frac{\partial}{\partial \theta}\left(\frac{v_\theta}{\sin\theta}\right) - \frac{1}{\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right)\right]$$

$$T^{2j}{}_{;j} = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{\partial}{\partial r}\left[\mu r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{\mu}{r}\frac{\partial v_r}{\partial \theta}\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[\frac{2\mu}{r}\left(\frac{\partial v_\theta}{\partial \theta} + v_r\right) + \left(\zeta - \frac{2}{3}\mu\right)\nabla\cdot\underline{v}\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left[\frac{\mu\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{\mu}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]$$

$$+ \frac{\mu}{r}\left[\frac{3}{r}\frac{\partial v_r}{\partial \theta} + 3r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{2\cot\theta}{r}\left(\sin\theta\frac{\partial}{\partial \theta}\left(\frac{v_\theta}{\sin\theta}\right) - \frac{1}{\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right)\right]$$

$$\text{in}\,\theta \quad T^{3j}_{\ ;j} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 T_{r\phi}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, T_{\theta\phi}\right) + \frac{1}{r\sin\theta}\frac{\partial T_{\phi\phi}}{\partial\phi} + \frac{1}{r}T_{r\phi} + \frac{\cot\theta}{r}T_{\theta\phi}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\mu\left(r\frac{\partial}{\partial r}(v_\phi/r) + \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right)\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\,\mu\left(\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\frac{v_\phi}{\sin\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right)\right]$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[-p + \frac{2\mu}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{2\mu v_r}{r} + \frac{2\mu v_\theta\cot\theta}{r} + \eta\right]$$

$$+ \frac{\mu}{r}\left[r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right] + \frac{\mu}{r}\left[\cot\theta\left(\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\frac{v_\phi}{\sin\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right)\right]$$

$$= \frac{\partial}{\partial r}\left[\mu r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{\mu}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right] + \frac{2\mu}{r}\left[r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right] + \frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{\mu\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{\mu}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right]$$

$$+ \frac{\mu}{r}\left[\frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{\cot\theta}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right] - \frac{1}{r\sin\theta}\frac{\partial p}{\partial\phi} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\frac{2\mu}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{2\mu v_r}{r} + \frac{2\mu v_\theta\cot\theta}{r} + (\zeta - \tfrac{2}{3}\mu)\underset{\sim}{\nabla}\cdot\underset{\sim}{v}\right]$$

$$+ \frac{\mu}{r}\left[r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{\cot\theta}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right]$$

$$\frac{\mu}{r}\ \text{term} = \frac{\mu}{r}\left[\overset{\checkmark}{2\cancel{\mu}r\frac{\partial}{\partial r}}\left(\frac{v_\phi}{r}\right) + \overset{\checkmark}{\frac{2}{r\sin\theta}}\frac{\partial v_r}{\partial\phi} + \overset{\checkmark}{\frac{\cos\theta}{r}}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \overset{\checkmark}{\frac{\cot\theta}{r\sin\theta}}\frac{\partial v_\theta}{\partial\phi} + \overset{\checkmark}{r\frac{\partial}{\partial r}}\left(\frac{v_\phi}{r}\right) + \overset{-}{\frac{\cos\theta}{r}}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \overset{-}{\frac{1}{r\sin\theta}}\frac{\partial v_r}{\partial\phi} + \frac{\cot\theta}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right]$$

$$= \frac{\mu}{r}\left[3r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{3}{r\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{2\cos\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{2\cot\theta}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right]$$

$$T^{3j}_{\ ;j} = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial\phi} + \frac{\partial}{\partial r}\left[\mu r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{\mu}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right] + \frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{\mu\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{\mu}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\frac{2\mu}{r}\left(\frac{1}{\sin\theta}\frac{\partial v_\phi}{\partial\phi} + v_r + v_\theta\cot\theta\right) + (\zeta - \tfrac{2}{3}\mu)\underset{\sim}{\nabla}\cdot\underset{\sim}{v}\right]$$

$$+ \frac{\mu}{r}\left[3r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{3}{r\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{2\cot\theta}{r}\left(\sin\theta\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right)\right]$$

26.10

$$\nabla\cdot\underset{\sim}{V} = V^i_{;i} = \frac{\partial v_r}{\partial r} + \frac{\partial}{\partial\theta}\left(\frac{v_\theta}{r}\right) + \frac{\partial}{\partial\phi}\left(\frac{v_\phi}{r\sin\theta}\right) + \frac{2v_r}{r} + \frac{\cot\theta\, v_\theta}{r}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{\cot\theta\, v_\theta}{r}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{r\sin\theta\,\partial\theta}\partial\left(\sin\theta\, v_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$T^{ij}_{;j} = \frac{\partial}{\partial r}\left[2\mu\frac{\partial v_r}{\partial r} - \frac{2\mu}{3}\left(\frac{\partial v_r}{\partial r} + \frac{2v_r}{r}\right)\right] + 4\mu\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)$$

$$= \frac{\partial}{\partial r}\left[\frac{4}{3}\mu\frac{\partial v_r}{\partial r} - \frac{4}{3}\frac{\mu v_r}{r}\right] + 4\mu\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)$$

$$= \frac{\partial}{\partial r}\left[\frac{4\mu r}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] + 4\mu\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)$$

$$= \frac{\partial}{\partial r}\left[\frac{4\mu r}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{4\mu r^3}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] - \frac{r^3}{r^2}\frac{\partial}{\partial r}\left[\frac{4}{3}\mu\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right]$$

$$= \frac{\partial}{\partial r}\left[\frac{4\mu r}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{4\mu r^3}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] - \frac{r\partial}{\partial r}\left[\frac{4\mu}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right]$$

but $$\frac{\partial}{\partial r}\left[\frac{4\mu r}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] - r\frac{\partial}{\partial r}\left[\frac{4\mu}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] = \frac{4\mu}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)$$

and $$\frac{4\mu}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{4\mu r^3}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] = \frac{1}{r^3}\frac{\partial}{\partial r}\left[\frac{4\mu r^4}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right]$$

26.10

27.30

$$\Phi = 2\mu\, E_{ij} E^{ij} + (\zeta - \tfrac{2}{3}\mu)\,(\nabla \cdot \underset{\sim}{v})^2$$

$$\Phi = 2\mu \left[E_{rr}E^{rr} + E_{\theta\theta}E^{\theta\theta} + E_{\phi\phi}E^{\phi\phi} + 2E_{r\theta}E^{r\theta} + 2E_{r\phi}E^{r\phi} + 2E_{\theta\phi}E^{\theta\phi} \right]$$

$$+ (\zeta - \tfrac{2}{3}\mu)(\nabla \cdot \underset{\sim}{v})^2$$

$$= 2\mu \Big\{ \left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + \left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot\theta}{r}\right)^2 + \frac{1}{2}\left(\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right)^2$$

$$+ \frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{\partial v_\phi}{\partial r} - \frac{v_\phi}{r}\right)^2 + \frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{1}{r}\frac{\partial v_\phi}{\partial \theta} - \frac{v_\phi \cot\theta}{r}\right)^2 \Big\}$$

$$+ (\zeta - \tfrac{2}{3}\mu)\left(\frac{\partial v_r}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{2v_r}{r} + \frac{\cot\theta}{r}v_\theta\right)^2$$

$$\Phi = 2\mu \Big\{ \left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + \left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot\theta}{r}\right)^2 + \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]^2$$

$$+ \frac{1}{2}\left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]^2 + \frac{1}{2}\left[\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right)\right]^2 \Big\}$$

$$+ (\zeta - \tfrac{2}{3}\mu)\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right]^2$$

1-D $\zeta = 0$ $\left(\frac{\partial}{\partial \theta} = \frac{\partial}{\partial \phi} = 0,\ v_\phi = v_\theta = 0 \quad \zeta = 0\right)$

$$\Phi = 2\mu \left\{ \left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{v_r}{r}\right)^2 + \left(\frac{v_r}{r}\right)^2 \right\} - \frac{2}{3}\mu \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r)\right]^2$$

$$= 2\mu \left\{ \left(\frac{\partial v_r}{\partial r}\right)^2 + \frac{2v_r^2}{r^2} \right\} - \frac{2}{3}\mu \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r)\right]^2 = \{ \quad 3 - \frac{2}{3}\mu \left[\frac{\partial v_r}{\partial r} + \frac{2v_r}{r}\right]^2$$

$$= 2\mu\left(\frac{\partial v_r}{\partial r}\right)^2 + \frac{4\mu v_r^2}{r^2} - \frac{2}{3}\mu \left[\left(\frac{\partial v_r}{\partial r}\right)^2 + 4\left(\frac{v_r}{r}\right)^2 + \frac{4v_r}{r}\frac{\partial v_r}{\partial r}\right]$$

$$= \frac{4}{3}\mu\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{4}{3}\mu\frac{v_r^2}{r^2}\right) - 2\cdot\frac{4}{3}\mu\frac{v_r}{r}\frac{\partial v_r}{\partial r} = \frac{4\mu}{3}\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \frac{v_r^2}{r^2} - \frac{2v_r}{r}\frac{\partial v_r}{\partial r}\right] = \frac{4\mu}{3}\left[r\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right]^2$$

$\boxed{27.33}$ $(\rho e + \frac{1}{2}\rho v^2)_{,t} + [\rho(e + v^2/2)v^j - v_i T^{ij} + q^j]_{;j} = v_i f^i$

for spherical flow with $f = 0$:

$$q^r_{;r} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k \frac{\partial T}{\partial r}\right)$$

$$[\rho(e + v^2/2)v^r]_{;r} = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 v_r \rho(e + v^2/2)]$$

$$v_r T^{11} = -v_r p + 2\mu v_r \frac{\partial v_r}{\partial r} - \frac{2}{3}\mu v_r\left(\frac{\partial v_r}{\partial r} + \frac{2 v_r}{r}\right)$$

$$= -p v_r + \frac{4}{3}\mu v_r \frac{\partial v_r}{\partial r} - \frac{4}{3}\mu v_r \cdot \frac{v_r}{r}$$

$$= -p v_r + \frac{4}{3}\mu v_r\left[\frac{\partial v_r}{\partial r} - \frac{v_r}{r}\right] = -p v_r + \frac{4}{3}\mu v_r\left[r\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right]$$

$$(v_r T^{11})_{;r} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r T^{11})$$

hence

$$\frac{\partial}{\partial t}(\rho e + \frac{1}{2}\rho v^2) + \frac{1}{r^2}\frac{\partial}{\partial r}\Big\{r^2 v_r \rho(e + v^2/2) + r^2 v_r\left[+p - \frac{4}{3}\mu r\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right]$$
$$- r^2 k \frac{\partial T}{\partial r}\Big\} = v_r f_r$$

27.36 $F = -\frac{GM\rho}{r^2} = -g\rho$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2 \rho v_r\left[e + \frac{p}{\rho} - \frac{v^2}{2} - \frac{4}{3}\nu r\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] - r^2 k \frac{dT}{dr}\right\} = -v_r \frac{GM\rho}{r^2}$$

$$\frac{\partial}{\partial r}\left\{\overbrace{(4\pi\rho r^2 v_r)}^{\dot{m}}\left[h - \frac{v^2}{2} - \frac{4}{3}\nu r\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] - 4\pi r^2 k \frac{dT}{dr}\right\} = -v_r g\rho \cdot 4\pi r^2 /$$
$$= -\dot{m}\, GM/r^2$$

$$\Rightarrow 4\dot{m}\left[h - \frac{v^2}{2} - \frac{4}{3}\nu r\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] - 4\pi r^2 k \frac{dT}{dr} = \cancel{-\dot{m}\int dr/r^2} - \dot{m}GM\int dr/r^2$$
$$= +\frac{\dot{m}GM}{r} + \text{constant}$$

✓

$\boxed{32,32}$

$$32.29: \quad \mathcal{D}f_0 = \frac{\partial f_0}{\partial N}\frac{DN}{Dt} + \frac{\partial f_0}{\partial v^i}\frac{Dv^i}{Dt} + \frac{\partial f_0}{\partial T}\frac{DT}{Dt} + U^i\left[\frac{\partial f_0}{\partial N}\frac{\partial N}{\partial x^i} + \frac{\partial f_0}{\partial V^j}\frac{\partial v^j}{\partial x^i} + \frac{\partial f_0}{\partial T}\frac{\partial T}{\partial x^i}\right] + a^i\frac{\partial f_0}{\partial u^i}$$

Use $\dfrac{\partial f_0}{\partial N} = \dfrac{f_0}{N}$

$$\frac{\partial f_0}{\partial v^i} = \left(\frac{m}{kT}\right)U^i f_0$$

$$\frac{\partial f_0}{\partial u^i} = -\frac{mU^i}{kT} f_0$$

$$\frac{\partial f_0}{\partial T} = \left[\frac{mU^2}{2kT} - \frac{3}{2}\right]\frac{f_0}{T}$$

$$\frac{DN}{Dt} = -N v^i{}_{,i} \qquad \frac{Dv_i}{Dt} = a_i - \frac{1}{\rho}P_{,i}$$

$$\begin{cases} \Rightarrow \dfrac{DT}{Dt} = -\left(\dfrac{2m}{3k}\right)\dfrac{P}{\rho} V^i{}_{,i} \\ \dfrac{De}{Dt} = \left(\dfrac{3k}{2m}\right)\left(\dfrac{DT}{Dt}\right) = -\dfrac{P}{\rho}V^i{}_{,i} \end{cases}$$

$$\mathcal{D}f_0 = \frac{f_0}{N}\left(-Nv^i{}_{,i}\right) + \left(\frac{m}{kT}\right)U^i f_0\left(a_i - \rho^{-1}P_{,i}\right) + \left[\frac{mU^2}{2kT} - \frac{3}{2}\right]\frac{f_0}{T}\left(-\frac{2m}{3k}\frac{P}{\rho}\right)V^i{}_{,i}$$

$$+ U^i\left[\frac{f_0}{\cancel{Nk}}\left(\frac{P_{,i}}{P} - \frac{T_{,i}}{T}\right) + f_0\left(\frac{m}{kT}\right)U^j v^j{}_{,i} + \left(\frac{mU^2}{2kT} - \frac{3}{2}\right)\frac{f_0}{T}T_{,i}\right] - \frac{mU^i}{kT}f_0 a^i$$

$$f_0^{-1}\mathcal{D}f_0 = -V^i{}_{,i} + \cancel{\frac{mU^i}{kT}a_i} - \frac{mU^i}{kT\rho}P_{,i} - \left[\frac{mU^2}{2kT} - \frac{3}{2}\right]\frac{2}{3}\left(\frac{P}{\rho m^{-1}kT} \equiv 1\right)V^i{}_{,i}$$

$$+ U^i\left[\frac{P_{,i}}{P} - \frac{T_{,i}}{T}\right] + \frac{m}{kT}U^i U^j V^j{}_{,i} + \left[\frac{mU^2}{2kT} - \frac{3}{2}\right]\frac{T_{,i}}{T}U^i - \cancel{\frac{mU^i a^i}{kT}}$$

$$= -\cancel{V^i{}_{,i}} - \frac{mU^i}{\rho kT}P_{,i} - \frac{mU^2}{3kT}V^i{}_{,i} + \cancel{V^i{}_{,i}} + U^i\frac{P_{,i}}{P} - U^i\frac{T_{,i}}{T} + \frac{mU^j}{kT}V^j{}_{,i} + \frac{mU^2}{2kT}\frac{T_{,i}}{T} - \frac{3}{2}\frac{T_{,i}}{T}$$

$$\frac{m}{\rho kT} = \frac{1}{P} \qquad = -\cancel{\frac{U^i P_{,i}}{P}} - \frac{mU^2}{3kT}V^i{}_{,i} + \cancel{\frac{U^i P_{,i}}{P}} - \frac{U^i T_{,i}}{T} + \frac{mU^i U^j}{kT}V^j{}_{,i} + \frac{mU^2 U^i}{2kT}\frac{T_{,i}}{T} - \frac{3}{2}\frac{T_{,i}}{T}U^i$$

$$= -\frac{mU^2}{3kT}V^i{}_{,i} + \frac{mU^i U^j}{kT}V^j{}_{,i} - \frac{U^i T_{,i}}{T} - \frac{3}{2}U^i\frac{T_{,i}}{T} + \frac{mU^2 U^i}{2kT}\frac{T_{,i}}{T}$$

$$f_0^{-1}\mathcal{D}f_0 = \left(-\frac{mU^2}{3kT}\delta^i_j + \frac{mU^iU^j}{kT}\right)V^j_{;i} + \left(-\frac{5}{2}U^i + \frac{mU^2U^i}{2kT}\right)\frac{T_{;i}}{T}$$

$$= \frac{m}{kT}\left\{-\frac{1}{3}U^2\delta^i_j + U^iU^j\right\}V^j_{;i} + \left(-\frac{5}{2} + \frac{mU^2}{2kT}\right)U^i(\ln T)_{;i}$$

32.36 & 32.37

$$\mathcal{I}(\phi_1) = -\frac{1}{N^2}\iint[\phi_1(u')+\phi_1(u_1')-\phi_1(u)-\phi_1(u_1)]f_0 f_0\, g\sigma\, d\Omega\, d^3u_1$$

$$J(\phi_1) = \iint[\quad]\ldots \qquad \mathcal{I}(\phi_1) = -\frac{1}{N^2}J(\phi_1) \qquad (32.22)$$

$$-N^2\mathcal{I}(\Phi_1) = \iint[\phi_1(\underset{\sim}{u}')+\phi_1(\underset{\sim}{u}_1')-\phi_1(\underset{\sim}{u})-\phi_1(\underset{\sim}{u}_1)]\, f_0(\underset{\sim}{u})f_0(\underset{\sim}{u}_1)\, g\sigma(\underset{\sim}{\Omega})\, d\Omega\, d^3u_1 = \mathcal{D}f_0$$

Where $f_0 \equiv N\left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-mu^2/2kT\right)$ (32.11)

(32.33)

now $$\mathcal{D}f_0 = \left\{U_i\left[\frac{mU^2}{2kT}-\frac{5}{2}\right](\ln T)_{,i} + \left(\frac{m}{kT}\right)(U_iU_j - \tfrac{1}{3}U^2\delta_{ij})V_{i,j}\right\}f_0$$
$$= -N^2\mathcal{I}(\phi_1)$$

(see notes for check of derivation)

We assume trial form for $\phi_1(\underset{\sim}{u})$:

$$\Phi_1 = -N^{-1}\left[\left(\frac{2kT}{m}\right)^{1/2}A_i\,(\ln T)_{,i} + B_{ij}D_{ij} + \Psi\right] \qquad (33.35)$$

And substitute this into

$$\mathcal{D}f_0 = -N^2\mathcal{I}(\Phi_1)$$

$$-N^2\mathcal{I}(\Phi_1) = -N^{-1}\left(\frac{2kT}{m}\right)^{1/2}(\ln T)_{,i}\iint[A_i(\underset{\sim}{u}')+A_i(\underset{\sim}{u}_1')-A_i(\underset{\sim}{u})-A_i(\underset{\sim}{u}_1)]\,f_0(\underset{\sim}{u})f_0(\underset{\sim}{u}_1)\cdot g\sigma\, d\Omega\, d^3u_1$$
$$-N^{-1}D_{ij}\iint[B_{ij}(\underset{\sim}{u}')+B_{ij}(\underset{\sim}{u}_1')-B_{ij}(\underset{\sim}{u})-B_{ij}(\underset{\sim}{u}_1)]\,f_0(\underset{\sim}{u})f_0(\underset{\sim}{u}_1)\,g\sigma\, d\Omega\, d^3u_1$$
$$-N^{-1}\iint[\Psi(\underset{\sim}{u}')+\Psi(\underset{\sim}{u}_1')\ \Psi(\underset{\sim}{u})-\Psi(\underset{\sim}{u}_1)]\,f_0(\underset{\sim}{u})f_0(\underset{\sim}{u}_1)\,g\sigma\, d\Omega\, d^3u_1$$

or
$$-N^2\mathcal{I}(\Phi_1) = -N^{-1}\left(\frac{2kT}{m}\right)^{1/2}(\ln T)_{,i}\, J(A_i) - N^{-1}D_{ij}\,J(B_{ij}) - N^{-1}J(\Psi)$$

where we have defined $J(\phi_*) = \iint[\phi(u')+\phi_1(u_1')-\phi_1(u)-\phi_1(u_1)]f_0(u)f_0(u_1)\ldots$
$= -N^2\mathcal{I}(\phi_*)$

Then $$-N^2\mathcal{I}(\phi_1) = -N^{-1}\left(\frac{2kT}{m}\right)^{1/2}(\ln T)_{,i}\left[-N^2\mathcal{I}(A_i)\right] - N^{-1}D_{ij}\left[-N^2\mathcal{I}(B_{ij})\right] - N^{-1}\left[N^2\mathcal{I}(\Psi)\right]$$

Thus

$$-N^2\mathcal{I}(\Phi_1) = +N\left(\frac{2kT}{m}\right)^{1/2}(\ln T)_{,i}\,\mathcal{I}(A_i) + N D_{ij}\,\mathcal{I}(B_{ij}) + N\,\mathcal{I}(4)$$

$$= \mathcal{D}f_0 = U_i\left[\frac{mU^2}{2kT} - \frac{5}{2}\right](\ln T)_{,i} f_0 + \left(\frac{m}{kT}\right)\left(U_iU_j - \tfrac{1}{3}U^2\delta_{ij}\right)D_{ij}f_0 + O \qquad (32.33)$$

$\Rightarrow$

(1) $N\left(\frac{2kT}{m}\right)^{1/2}\cancel{(\ln T)_{,i}}\,\mathcal{I}(A_i) = U_i\left[\frac{mU^2}{2kT} - \frac{5}{2}\right]\cancel{(\ln T)_{,i}}f_0$

(2) $N\cancel{D_{ij}}\,\mathcal{I}(B_{ij}) = \left(\frac{m}{kT}\right)\left(U_iU_j - \tfrac{1}{3}U^2\delta_{ij}\right)\cancel{D_{ij}}f_0$

(3) $N\mathcal{I}(4) = 0$

or

(1) $N\,\mathcal{I}(A_i) = \left(\frac{m}{2kT}\right)^{1/2}U_i\left[\frac{mU^2}{2kT} - \frac{5}{2}\right]f_0$

(2) $N\mathcal{I}(B_{ij}) = \left(\frac{m}{kT}\right)\left(U_iU_j - \tfrac{1}{3}U^2\delta_{ij}\right)f_0 = 2\left(\frac{m}{2kT}\right)\left(U_iU_j - \tfrac{1}{3}U^2\delta_{ij}\right)f_0$

(3) $\mathcal{I}(4) = 0$

Rescale the velocities; define $\mathcal{U}_i = \left(\frac{m}{2kT}\right)^{1/2}U_i$, $\mathcal{U} = \left(\frac{m}{2kT}\right)^{1/2}U$, (32,39) then

(1) $N\,\mathcal{I}(A_i) = \mathcal{U}_i\left[\mathcal{U}^2 - \frac{5}{2}\right]f_0$ (32.36)

(2) $N\,\mathcal{I}(B_{ij}) = 2\left(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}\mathcal{U}^2\delta_{ij}\right)f_0$ (32.37)

(3) $\mathcal{I}(4) = 0$ (32.38)

$$A_i = A(\mathcal{U},T)\,\mathcal{U}_i \qquad (32.40)$$

or $\underset{\sim}{A} = A(\mathcal{U},T)\,\underset{\sim}{\mathcal{U}}$

Now use (32.19)-(32.21): $\int f_i\, d^3u = 0$; $\int \underset{\sim}{u} f_i\, d^3u = 0$; $\int U^2 f_i\, d^3u = 0$ and $f_1 = f_0 \phi_1$, to obtain:

$$\int f_0 \phi_1\, d^3U = 0 \qquad (32.43)$$

$$\int U_i f_0 \phi_1\, d^3U = 0 \qquad (32.44)$$

$$\int U^2 f_0\, \phi_1\, d^3U = 0 \qquad (32.45)$$

From (32.40) - (32.41) + (32.35)

$$\phi_1 = -N^{-1}\left\{\left(\frac{2kT}{m}\right)^{1/2} A(\mathcal{U},T)\,\mathcal{U}_i\,(\ln T)_{,i} + D_{ij} : B(\mathcal{U},T)\,(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}\mathcal{U}^2\delta_{ij}) + \alpha_1 + \underset{\sim}{\alpha}_2 \cdot \underset{\sim}{U} + \alpha_3 U^2\right\}$$

Integral $\int f_0 \Phi_1\, d^3U$ becomes:

$$0 = \cancel{-N}\int f_0\left[\left(\frac{2kT}{m}\right)^{1/2}(\ln T)_{;i}\, A(\mathcal{U},T)\,\mathcal{U}_i + D_{ij}\, B(\mathcal{U},T)(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}\mathcal{U}^2\delta_{ij}) + \alpha_1 + \underset{\sim}{\alpha}_2 \cdot \underset{\sim}{U} + \alpha_3 U^2\right] d^3U$$

$$= \left(\frac{2kT}{m}\right)^{1/2}(\ln T)_{;i} \int f_0\, A(\mathcal{U},T)\,\mathcal{U}_i\, d^3\mathcal{U} + D_{ij}\int B(\mathcal{U},T)(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}\mathcal{U}^2\delta_{ij})\, f_0\, d^3\mathcal{U}$$

$$+ \alpha_1 \int f_0\, d^3U + \underset{\sim}{\alpha}_2 \cdot \int \underset{\sim}{U} f_0\, d^3U + \alpha_3 \int f_0\, U^2 d^3U$$

$$= (\ln T)_{;i}\int \cancelto{0}{f_0 A(\mathcal{U},T) U_i\, d^3U} + D_{ij}\int \cancelto{0}{B(\mathcal{U},T)(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}\mathcal{U}^2\delta_{ij}) f_0\, d^3U}$$

$$+ \int \alpha_1 f_0\, d^3U + \int \underset{\sim}{\alpha}_2 \cdot \underset{\sim}{U} f_0\, d^3U + \int \alpha_3 f_0\, U^2 d^3U$$

$$0 = \int (\alpha_1 + \alpha_3 U^2) f_0\, d^3U \qquad (32.46)$$

Integral $\int U_i f_0 \Phi_1 d^3U = 0$ becomes:

$$0 = \cancel{-N^{-1}}\Big\{ \int d^3U f_0 \left(\frac{2kT}{m}\right)^{1/2} A(\mathcal{U},T)\mathcal{U}_i (\ln T)_{,i}\, U_i + \int d^3U f_0 D_{ij} \cdot B(\mathcal{U},T) \overset{0}{\cancel{(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}U\delta_{ij})U_i}}$$

$$+ \int d^3U f_0 (\alpha_1 + \underset{\sim}{\alpha_2} \cdot \underset{\sim}{U} + \alpha_3 U^2) U_i \Big\}$$

$$= \int d^3U f_0 U_i^2 A(\mathcal{U},T)(\ln T)_{,i} + \int d^3U f_0 (\alpha_1 + \alpha_{2j} U_j + \alpha_3 U^2) U_i$$

2nd integral is non-zero only when $j = i$, hence

$$0 = (\ln T)_{,i} \int A(\mathcal{U},T) U_i^2 f_0 d^3U + \alpha_{2i} \int U_i^2 f_0 d^3U$$

$0 =$ [illegible]

$$0 = \int [\alpha_{2i} + A(\mathcal{U},T)(\ln T)_{,i}] f_0 U^2 d^3U \qquad (32.47)$$

Integral $\int U^2 f_0 \Phi_1 d^3U = 0$ becomes:

$$0 = \cancel{-N^{-1}}\Big\{ \int d^3U f_0 \left(\frac{2kT}{m}\right)^{1/2} A(\mathcal{U},T)\mathcal{U}_i (\ln T)_{,i}\, U_i^2 + \int d^3U f_0 D_{ij} \cdot B(\mathcal{U},T)(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}U^2\delta_{ij})U_i^2$$

$$+ \int d^3U f_0 (\alpha_1 + \underline{\alpha_2} \cdot \underline{U} + \alpha_3 U^2) U^2 \Big\}$$

$$= \int d^3U f_0 A(\mathcal{U},T)\overset{0}{\cancel{(\ln T)_{,i} U_i^3}} + \int d^3U f_0 D_{ij} B(\mathcal{U},T)\overset{0}{\cancel{(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}U^2\delta_{ij})U_i^2}}$$

$$+ \int [\alpha_1 f_0 U^2 + f_0 \alpha_{2i} U_i U^2 + \alpha_3 f_0 U^4] d^3U$$

$$0 = \int [\alpha_1 U^2 + \alpha_3 U^4] f_0 d^3U \qquad (32.48)$$

$$q_k = \frac{m}{2}\int U_k U^2 f_0 \Phi_1 \, d3U$$

$$= -\frac{m}{N2}\int f_0 \, d3U \, U_k U^2 \left\{ \left(\frac{2kT}{m}\right)^{1/2} A(\mathcal{U},T)\mathcal{U}_i (\ln T)_{,i} + D_{ij} B(\mathcal{U},T)\left(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}\mathcal{U}^2\delta_{ij}\right) \right\}$$

$$= -\frac{m}{2N}\int f_0 \, d3U \, U^2 U_k \left\{ A(\mathcal{U},T)(\ln T)_{,i} U_i + D_{ij} B(\mathcal{U},T)\overset{0}{\cancel{\left(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}\mathcal{U}^2\delta_{ij}\right)}} \right\}$$

$$= -\frac{m}{2N}(\ln T)_{,i}\int f_0 \, d3\mathcal{U} \, A(\mathcal{U},T) U_i U_k U^2$$

Now

$$f_0 = N\left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-mu^2/2kT\right) = N\left(\frac{m}{2kT}\right)^{3/2}\pi^{-3/2}\exp\left[-\mathcal{U}^2\right]$$

$$\text{and } f_0 \, d^3U = N\pi^{-3/2} e^{-\mathcal{U}^2} d^3\mathcal{U} \equiv N \mathfrak{f}_0 \, d^3\mathcal{U}$$

$$\text{and thus } \quad \mathfrak{f}_0 \equiv \pi^{-3/2} e^{-\mathcal{U}^2} \qquad (32.52)$$

Then

$$q_k = -\frac{m}{2N}\cdot N\left[\int \mathfrak{f}_0 \, d^3\mathcal{U} \, A(\mathcal{U},T) U^2 U_i U_k\right](\ln T)_{,i}$$

$$= -\frac{m}{2}(\ln T)_{,i}\int \mathfrak{f}_0 \, d^3\mathcal{U} \, A(\mathcal{U},T) U_i U_k U^2 \qquad U^2 = \left(\frac{2kT}{m}\right)\mathcal{U}^2$$

now scale U_i and U_k and U^2 by $\left(\frac{m}{2kT}\right)^{1/2}$ $\quad U_i U_k = \left(\frac{2kT}{m}\right)\left(\frac{m}{2kT}\right)U_i U_k = \left(\frac{2kT}{m}\right)\mathcal{U}_i\mathcal{U}_k$

$$q_k = -\frac{2(kT)^2}{m}(\ln T)_{,i}\int \mathfrak{f}_0 \, d^3\mathcal{U} \, A(\mathcal{U},T)\mathcal{U}_i\mathcal{U}_k\mathcal{U}^2$$

$$= -\frac{2k^2T}{m} : T_{,i}\int \mathfrak{f}_0 \, d^3\mathcal{U} \, A(\mathcal{U},T)\mathcal{U}_i\mathcal{U}_k\mathcal{U}^2$$

$$= -\frac{2k^2T}{m} T_{,i} \, \delta_{ik}\int f_0 \, d^3\mathcal{U} \, A(\mathcal{U},T)\mathcal{U}_i\mathcal{U}_k\mathcal{U}^2$$

$$q_k = -\left[\frac{2k^2T}{m}\int f_0 \, d^3\mathcal{U} \, A(\mathcal{U},T)\mathcal{U}_i^2\mathcal{U}^2\right]T_{,k} \qquad (32.55)$$

U_i

$d(\cos\theta) = -\sin\theta\, d\theta$

$\cos^2\theta\, d(\cos\theta) = \frac{\cos^3\theta}{3}$

~~$d(\sin\theta) = \cos\theta\, d\theta$~~

~~$\int \sin\theta\, d(\sin\theta) = \frac{\sin^2\theta}{2}$~~

$$\int \mathcal{U}_\ell^{\,2}\,\mathcal{U}^2 A(\mathcal{U},T)\, f_0(\mathcal{U})\, \mathcal{U}^2 d\mathcal{U}\, \sin\theta\, d\theta\, d\phi$$

and choose $\mathcal{U}_\ell$ so that $\mathcal{U}_\ell = \mathcal{U}\cos\phi$, then

$$\int \mathcal{U}^4 A(\mathcal{U},T)\, f_0(\mathcal{U})\, \mathcal{U}^2 d\mathcal{U} \int \cos^2\theta \sin\theta\, d\theta\, d\varphi$$

$$= \left(\frac{2}{3}\right) 2\pi \int \mathcal{U}^6 A(\mathcal{U},T)\, f_0(\mathcal{U})\, d\mathcal{U} = \frac{4}{3}\pi \int \mathcal{U}^6 A(\mathcal{U},T)\, f_0(\mathcal{U})\, d\mathcal{U}$$

and the thermal conductivity K is:

$$K = \frac{2k^2T}{m}\cdot\int = \frac{8}{3}\frac{\pi k^2 T}{m}\int \mathcal{U}^6 A(\mathcal{U},T)\, f_0(\mathcal{U})\, d\mathcal{U} \quad ; \qquad (32.54)$$

$$q_k = -K\,T_{,k}$$

* * *

$$\sigma_{ij}' = -m\int U_i U_j f_0 \Phi_1\, d^3U$$

$$= -m\int U_i U_j \left[\cancel{N}\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\mathcal{U}^2}\right]\left[-N^{-1} D_{k\ell} B(\mathcal{U},T)\left(\mathcal{U}_k\mathcal{U}_\ell - \frac{1}{3}\mathcal{U}^2\delta_{k\ell}\right)\right] d^3U$$

$$= +m\int U_i U_j \left(\pi^{-3/2} e^{-\mathcal{U}^2}\right) d^3\mathcal{U}\, D_{k\ell}\left(\mathcal{U}_k\mathcal{U}_\ell - \frac{1}{3}\mathcal{U}^2\delta_{k\ell}\right) B(\mathcal{U},T)$$

$$= +m\left(\frac{2kT}{m}\right)\int f_0\, d^3\mathcal{U}\, \mathcal{U}_i\mathcal{U}_j \left(\mathcal{U}_k\mathcal{U}_k - \frac{1}{3}\mathcal{U}^2 \overset{0}{\cancel{\delta_{k\ell}}}\right) D_{k\ell} B(\mathcal{U},T)$$

$$= 2kT\int f_0\, d^3\mathcal{U}\, \mathcal{U}_i^{\,2}\mathcal{U}_j^{\,2}\, 2D_{ij}'\, B(\mathcal{U},T)$$

$$\mu = 2kT\int f_0\, d^3\mathcal{U}\, \mathcal{U}_i^{\,2}\mathcal{U}_j^{\,2} B(\mathcal{U},T)$$

([illegible]) $\sin\theta \, d\theta \, d\phi$

$= \frac{2}{3} - \frac{2}{5}$

$\cos^2\theta \, (1 - \cos^2\theta) \sin\theta \, d\theta$ $\quad = \frac{10-6}{15} = \frac{4}{15}$

$= [\cos^2\theta - \cos^4\theta] \sin\theta \, d\theta$

$= [(\cos^2 - \cos^4) \, d(\cos\theta)] = \frac{\cos^3}{3} - \frac{\cos^5}{5}$

let $U_i = U\cos\theta \quad U_j = U\sin\theta\cos\phi \quad d^3U = U^2 \, dU \sin\theta \, d\theta \, d\phi$

then

$$\mu = 2kT \int f_0 B(U,T) \, U^2 \cdot U^4 \, dU \int \cos^2\theta \sin^2\theta \sin\theta \, d\theta \int \cos^2\phi \, d\phi$$

$$= 2kT \int f_0(U) \, B(U,T) \, U^6 \, dU \underbrace{\int \cos^2\theta \sin^3\theta \, d\theta}_{4/15} \int \underbrace{\cos^2\phi \, d\phi}_{\pi}$$

$$= \frac{8\pi kT}{15} \int f_0(U) \, B(U,T) \, U^6 \, dU \qquad (32.58)$$

$\delta(FI(F)) = \delta F I(F) + F I(\delta F)$ $\qquad = C\left[\int d^3\mathcal{U} F(2G - \mathcal{J}(F))\right]$

$H(= K \text{ or } \mu) = C\left[2\int FG\, d^3\mathcal{U} - \int F \mathcal{J}(F)\, d^3\mathcal{U}\right]$

F is the solution of $\mathcal{J}(F) = G$

Variation of H with small variations δF of F:

$$\delta H = 2C\int \delta F[G - \mathcal{J}(F)]\, d^3\mathcal{U} \qquad (33.3)$$

$$K = \frac{2k^2T}{3\cancel{k}m}\int \mathcal{U}_{\cancel{k}}^2\, \mathcal{U}^2 A(\mathcal{U},T) f_0(\mathcal{U})\, d^3\mathcal{U} - \frac{5}{2}\int \mathcal{U}^2 A(\mathcal{U},T) f_0(\mathcal{U})\, d^3\mathcal{U} \left(\frac{2k^2T}{3m}\right)$$

~~$K = \int \mathcal{U}^2 d^3\mathcal{U} A(\mathcal{U},T) f_0(u)\left[\frac{2k^2T}{m}\mathcal{U}_e^2 - \frac{5}{2}\right]$~~

$$K = \left(\frac{2k^2T}{3m}\right)\int A(\mathcal{U},T)\, \cancel{f_0}\, \mathcal{U}^2 f_0(u)\, [\mathcal{U}^2 - 5/2]\, d^3\mathcal{U}$$

$$= \left(\frac{2k^2T}{3m}\right)\int A_i \boxed{\mathcal{U}_i f_0(u) [\mathcal{U}^2 - 5/2]}\, d^3\mathcal{U} \qquad (33.10)$$

now (32.36)

$$N\mathcal{J}(A_i) = \mathcal{U}_i(\mathcal{U}^2 - 5/2)\cancel{f_0}\, f_0 = \mathcal{U}_i(\mathcal{U}^2 - 5/2) . N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\mathcal{U}^2}$$

$$= \mathcal{U}_i(\mathcal{U}^2 - 5/2)\, N\left(\frac{m}{2kT}\right)^{3/2} \boxed{\left(\pi^{-3/2} e^{-\mathcal{U}^2}\right)}$$

$$= \mathcal{U}_i(\mathcal{U}^2 - 5/2)\, N\left(\frac{m}{2kT}\right)^{3/2} f_0(u)$$

hence

$$\mathcal{J}(A_i) = \left(\frac{m}{2kT}\right)^{3/2} f_0(u)\, \mathcal{U}_i(\mathcal{U}^2 - 5/2)$$

$$\Longrightarrow \mathcal{U}_i f_0(\mathcal{U})(\mathcal{U}^2 - 5/2) = \left(\frac{2kT}{m}\right)^{3/2}\mathcal{J}(A_i)$$

and therefore

$$K = \left(\frac{2k^2T}{3m}\right) \int A_i \left(\frac{2kT}{m}\right)^{3/2} \mathcal{I}(A_i)\, d^3\mathcal{U}$$

no! nondimensionalizing $\mathcal{I}(A_0)$ gives a factor that cancels the $\left(\frac{m}{2kT}\right)^{3/2}$, hence

$$\mathcal{I}(A_i) = \mathcal{U}_i(\mathcal{U}^2 - 5/2)\, f_0(\mathcal{U})$$

and

$$K = \frac{2k^2T}{3m}\left\{2\int A_i\, \mathcal{U}_i(\mathcal{U}^2 - 5/2) f_0(\mathcal{U})\, d^3\mathcal{U} - \int A_i \mathcal{I}(A_i)\, d^3\mathcal{U}\right\}$$

$$= C\left[\int FG\, d^3\mathcal{U} - \int F\mathcal{I}(F)\, d^3\mathcal{U}\right]$$

with $F = A_i$, $G = \mathcal{U}_i(\mathcal{U}^2 - 5/2) f_0$, $C = \frac{2k^2T}{3m}$

Viscosity μ:

$$\mu = (8\pi kT/15)\int B(\mathcal{U},T)\,\mathcal{U}^6 f_0(\mathcal{U})\, d\mathcal{U}$$

$$= \frac{1}{4\pi}\left(\frac{8\pi kT}{15}\right)\int B(\mathcal{U},T)\,\mathcal{U}^4 f_0(\mathcal{U})\, d^3\mathcal{U}$$

$$= (2kT/15)\int B(\mathcal{U},T)\,\mathcal{U}^4 f_0(\mathcal{U})\, d^3\mathcal{U}$$

Now $\mathcal{U}_\ell^\circ \mathcal{U}_m \equiv \mathcal{U}_\ell \mathcal{U}_m - \frac{1}{3}\mathcal{U}^2 \delta_{\ell m}$ satisfies $(\mathcal{U}_\ell^\circ \mathcal{U}_m)(\mathcal{U}_\ell^\circ \mathcal{U}_m) = \frac{2}{3}\mathcal{U}^4$, hence

$$U^4 = \frac{3}{2}(U_e^\circ U_m)(U_e^\circ U_m)$$

and

$$\mu = \frac{3}{2}\left(\frac{2kT}{15}\right)\int B(U,T)(U_e^\circ U_m)(U_e^\circ U_m)\, f_o(u)\, d^3U \qquad (33.15a)$$

$$= (kT/5)\int B_{em}(U_e^\circ U_m)\, f_o(u)\, d^3U \qquad (33.15b)$$

Now $N\,\mathcal{J}(B_{em}) = 2(U_e^\circ U_m) f_o = N\,\mathcal{J}[B_{em}(U)]$

$$\Rightarrow N\,\mathcal{J}(B_{em}(u)) = 2(U_e^\circ U_m) N f_o$$

hence

$$\mathcal{J}(B_{em}(u)) = 2\, U_e^\circ U_m f_o$$

and

$$\mu = (kT/10)\int B_{em}\,\mathcal{J}(B_{em})\, d^3U \qquad (33.15c)$$

Thus (33.16)

$$\mu = (kT/10)\left[2\int B_{em}(2U_e^\circ U_m f_o)\, d^3U - \int B_{em}\,\mathcal{J}(B_{em})\, d^3U\right]$$

$$\mu = C\left[2\int F G\, d^3U - \int F\,\mathcal{J}(F)\, d^3U\right]$$

with $F = B_{em}$, $G = 2U_e^\circ U_m f_o$, $C = kT/10$

$$H = C\left[2\alpha \int F_0 G\, d^3u - \alpha^2 \int F_0\, J(F_0)\, d^3u\right]$$

$$\alpha = \int F_0 G\, d^3u \Big/ \int F_0 J\, d^3u$$

$$\Rightarrow H = C\left[2\frac{\left(\int F_0 G\, d^3u\right)^2}{\int F_0 J\, d^3u} - \left(\frac{\int F_0 G\, d^3u}{\int F_0 J\, d^3u}\right)^2 \int F_0 J\, d^3u\right]$$

Calculate K :

$$= C\left(\int F_0 G\, d^3u\right)^2 \Big/ \int F_0 J\, d^3u$$

Trial function $F = \alpha F_0$ $\quad F_0 = A_i^0 = A^0 u_i$

$$A^0 = (u^2 - 5/2) \qquad F_0 = u_i(u^2 - 5/2)$$

$$G = u_i(u^2 - 5/2) f_0$$

~~$K_1 = C_K \int$~~

$$K_1 = C_K \frac{\left[\int u_i(u^2-5/2)\, u_i(u^2-5/2) f_0\, d^3u\right]^2}{\int u_i(u^2-5/2)\, J\left[u_i(u^2-5/2)\right] d^3u}$$

$$= \left(\frac{2k^2T}{3m}\right)\frac{\left(\int u^2(u^2-5/2)^2 f_0\, d^3u\right)^2}{\int u_i(u^2-5/2)\, J\left[u_i(u^2-5/2)\right] d^3u} = \frac{2k^2T}{3m}\cdot\frac{I_1^2}{I_2}$$

$$I_1 = \int u^2(u^2-5/2)^2 \pi^{-3/2} e^{-u^2} \cdot 4\pi u^2\, du$$

$$= (4\pi\, \pi^{-3/2})\int u^4(u^2-5/2)^2 e^{-u^2}\, du = 4\pi^{-1/2}\int\left[u^8 - 5u^6 + \tfrac{25}{4}u^4\right]e^{-u^2}\, du$$

15×
7
1 5

$$= (4/\sqrt{\pi})\left[\int u^8 e^{-u^2} du - 5\int u^6 e^{-u^2} du + \tfrac{25}{4}\int u^4 e^{-u^2} du\right] = \frac{4}{\sqrt{\pi}}\left\{\frac{7\cdot5\cdot3}{2(2)^4}\pi^{1/2} - 5\frac{5\cdot3}{2(2)^3}\pi^{1/2} + \frac{25\cdot3}{4\;2(2)^2}\pi^{1/2}\right\}$$

$$= 4\left\{\frac{105}{32} - \frac{75}{16} + \frac{25}{32}\right\} = 4\left\{\frac{105}{32} - \frac{150}{32} + \frac{75}{32}\right\} = 4\left\{\frac{105-75}{32}\right\} = 4\left\{\frac{30}{32}\right\} = \frac{30}{8} = \frac{15}{4}$$

$$I_2 = \int$$

$$-N^2\,\mathcal{I}(\Phi_1) = \iint \left[\phi_1(\underset{\sim}{u}') + \phi_1(\underset{\sim}{u}_1') - \phi_1(\underset{\sim}{u}) - \phi_1(\underset{\sim}{u}_1)\right]$$

$$\cdot f_0(\underset{\sim}{u})\, f_0(\underset{\sim}{u}_1)\, g\, \sigma(\underset{\sim}{\Omega})\, d\Omega\, d^3u_1$$

$$= \iint [\quad]\, N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\mathcal{U}^2} \cdot N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\mathcal{U}_1^2} \cdot \gamma\left(\frac{2kT}{m}\right)^{1/2} \sigma\, d\Omega \left(\frac{2kT}{m}\right)^{3/2} d^3\mathcal{U}_1$$

$$= \iint [\,N^2\left(\pi^{-3/2} e^{-\mathcal{U}^2}\right)\left(\pi^{-3/2} e^{-\mathcal{U}_1'}\right) \cancel{\gamma_1} \cancel{\left(\frac{2kT}{m}\right)^{1/2}} \left(\frac{m}{2kT}\right)^{\overset{1}{\cancel{3/2}}} \cancel{\left(\frac{m}{2kT}\right)^{3/2}} \cancel{\left(\frac{2kT}{m}\right)^{3/2}} \gamma\sigma\, d\Omega\, d^3\mathcal{U}_1$$

$$\cancel{-N^2}\,\cancel{\mathcal{I}(\Phi_1)} = \cancel{N^2} \iint f_0(\mathcal{U})\, f_0(\mathcal{U}_1)\, [\quad] \left(\frac{m}{2kT}\right) \gamma\sigma\, d\Omega\, d^3\mathcal{U}_1$$

$$\mathcal{I}(\Phi_1) = \left(\frac{m}{2kT}\right) \iint \left[\phi_1(\underset{\sim}{u}) + \phi_1(\underset{\sim}{u}_1) - \phi_1(\underset{\sim}{u}') - \phi_1(\underset{\sim}{u}_1')\right] f_0(\mathcal{U})\, f_0(\mathcal{U}_1)\, \gamma\sigma\, d\Omega\, d^3\mathcal{U}_1$$

$$\phi_1(\underset{\sim}{u}) = U_i(U^2 - 5/2) = \left(\frac{2kT}{m}\right)^{3/2} \mathcal{U}_i(\mathcal{U}^2 - 5/2)$$ (ok because 5/2 was a hokey in there anyway!)

$$\mathcal{I}(\Phi_1) = \left(\frac{2kT}{m}\right)^{1/2} \iint \left[\phi_1(\underset{\sim}{\mathcal{U}}) + \phi_1(\underset{\sim}{\mathcal{U}}_1) - \phi_1(\underset{\sim}{\mathcal{U}}') - \phi_1(\underset{\sim}{\mathcal{U}}_1')\right] f_0(\mathcal{U})\, f_0(\mathcal{U}_1)\, \gamma\sigma\, d\Omega\, d^3\mathcal{U}_1$$

Now

$$I_2 = \iint d^3\mathcal{U}\, d^3\mathcal{U}_1 \int \sigma(\underset{\sim}{\Omega})\, d\Omega\, f_0(\mathcal{U})\, f_0(\mathcal{U}_1)\, \gamma \;\cdot$$

$$(\mathcal{U}^2 - 5/2)\,\underset{\sim}{\mathcal{U}} \cdot \left[\underset{\sim}{\mathcal{U}}(\mathcal{U}^2 - 5/2) + \underset{\sim}{\mathcal{U}}_1(\mathcal{U}_1^2 - 5/2) - \underset{\sim}{\mathcal{U}}'(\mathcal{U}'^2 - 5/2) - \underset{\sim}{\mathcal{U}}_1'(\mathcal{U}_1'^2 - 5/2)\right]$$

$$d^3\mathcal{U}\, d^3\mathcal{U}_1 = (4\pi\gamma^2 d\gamma)(4\pi\Gamma^2 d\Gamma)\,(d\omega_\gamma/4\pi)(d\omega_\Gamma/4\pi)$$

$$f_0(\mathcal{U})\, f_0(\mathcal{U}_1) = \pi^{-3}\exp(-\mathcal{U}^2-\mathcal{U}_1^2) = \pi^{-3}\exp[-2\Gamma^2-\tfrac{1}{2}\gamma^2]$$

$$\underset{\sim}{\mathcal{U}} = \underset{\sim}{\Gamma}+\tfrac{1}{2}\underset{\sim}{\gamma} \Rightarrow \mathcal{U}^2 = \Gamma^2+\tfrac{1}{4}\gamma^2+\underset{\sim}{\gamma}\cdot\Gamma$$

and $\mathcal{U}^2-5/2 = \Gamma^2+\gamma^2/4-5/2 \;+\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma}$

Drop $-5/2$ terms throughout square bracket and consider

$$\underset{\sim}{\mathcal{U}}\circ\left[\mathcal{U}^2\underset{\sim}{\mathcal{U}}+\mathcal{U}_1^2\underset{\sim}{\mathcal{U}}_1-\mathcal{U}'^2\underset{\sim}{\mathcal{U}}'-\underset{\sim}{\mathcal{U}}_1'^2\underset{\sim}{\mathcal{U}}_1'\right]$$

$$=\mathcal{U}^2\cdot\mathcal{U}^2+\mathcal{U}_1^2\underset{\sim}{\mathcal{U}}\cdot\underset{\sim}{\mathcal{U}}_1-\mathcal{U}'^2\underset{\sim}{\mathcal{U}}\cdot\underset{\sim}{\mathcal{U}}'-\mathcal{U}_1'^2\underset{\sim}{\mathcal{U}}\cdot\mathcal{U}_1'$$

$$=[[\underset{\sim}{\Gamma}+\underset{\sim}{\gamma}/2)\cdot(\underset{\sim}{\Gamma}+\underset{\sim}{\gamma}/2)]^2+[(\underset{\sim}{\Gamma}-\underset{\sim}{\gamma}/2)\cdot(\underset{\sim}{\Gamma}-\underset{\sim}{\gamma}/2)(\underset{\sim}{\Gamma}-\underset{\sim}{\gamma}/2)\cdot(\underset{\sim}{\Gamma}+\underset{\sim}{\gamma}/2)$$

$$-(\underset{\sim}{\Gamma}+\underset{\sim}{\gamma}'/2)\cdot(\underset{\sim}{\Gamma}+\underset{\sim}{\gamma}'/2)(\underset{\sim}{\Gamma}+\underset{\sim}{\gamma}/2)\cdot(\underset{\sim}{\Gamma}+\underset{\sim}{\gamma}/2)$$

$$-(\underset{\sim}{\Gamma}-\underset{\sim}{\gamma}'/2)\cdot(\underset{\sim}{\Gamma}-\underset{\sim}{\gamma}'/2)(\underset{\sim}{\Gamma}+\underset{\sim}{\gamma}/2)\cdot(\underset{\sim}{\Gamma}-\underset{\sim}{\gamma}/2)$$

$$=[\Gamma^2+\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma}+\gamma^2/4]^2+[\Gamma^2-\underset{\sim}{\gamma}\cdot\Gamma+\gamma^2/4][\Gamma^2-\gamma^2/4]$$

$$-[\Gamma^2+\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'+\gamma'^2/4][\Gamma^2+\underset{\sim}{\Gamma}\cdot(\underset{\sim}{\gamma}+\underset{\sim}{\gamma}')/2+\underset{\sim}{\gamma}\cdot\underset{\sim}{\gamma}'/4]$$

$$-[\Gamma^2-\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}+\gamma'^2/4][\Gamma^2+\underset{\sim}{\Gamma}\cdot(\underset{\sim}{\gamma}-\underset{\sim}{\gamma}')/2-\underset{\sim}{\gamma}\cdot\underset{\sim}{\gamma}'/4]$$

~~$=\Gamma^2(\Gamma^2+\gamma^2/4)+\Gamma^2(\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma})+\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma}[\Gamma^2+\gamma^2/4]+(\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma})^2+\gamma^2/4(\Gamma^2+\underset{\sim}{\gamma}\cdot\Gamma+\gamma^2/4)$~~

$$=[(\Gamma^2+\gamma^2/4)+\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma}][(\Gamma^2+\gamma^2/4)+\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma}]+[(\Gamma^2+\gamma^2/4)-\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma}][\Gamma^2-\gamma^2/4]$$

$$*-[\Gamma^2+\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'+\gamma'^2/4]\Gamma^2 \;*-[\Gamma^2+\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'+\gamma'^2/4]\underset{\sim}{\Gamma}\cdot(\underset{\sim}{\gamma}+\underset{\sim}{\gamma}')-[\Gamma^2+\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'+\gamma'^2/4](\underset{\sim}{\gamma}\cdot\underset{\sim}{\gamma}'/4)$$

$$-[\Gamma^2-\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'+\gamma'^2/4]\Gamma^2-[\Gamma^2+\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'+\gamma'^2/4]\underset{\sim}{\Gamma}\cdot(\underset{\sim}{\gamma}-\underset{\sim}{\gamma}')+[\Gamma^2-\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'+\gamma'^2/4](\underset{\sim}{\gamma}\cdot\underset{\sim}{\gamma}'/4)$$

$$u^2 u^2 + u_1^2\, \underset{\sim}{u}_1 \cdot \underset{\sim}{u} - u'^2\, \underset{\sim}{u} \cdot \underset{\sim}{u}' - \underset{\sim}{u}_1'^2\, \underset{\sim}{u} \cdot \underset{\sim}{u}_1'$$

$$= \left(\underset{\sim}{\Gamma} + \tfrac{1}{2}\underset{\sim}{\gamma}\right)^2 \left(\underset{\sim}{\Gamma} + \tfrac{1}{2}\underset{\sim}{\gamma}\right) + \left(\underset{\sim}{\Gamma} - \tfrac{1}{2}\underset{\sim}{\gamma}\right)^2 \left(\underset{\sim}{\Gamma} - \tfrac{1}{2}\underset{\sim}{\gamma}\right)$$

$$- \left(\underset{\sim}{\Gamma} + \tfrac{1}{2}\underset{\sim}{\gamma}'\right)^2 \cdot \left(\underset{\sim}{\Gamma} + \tfrac{1}{2}\underset{\sim}{\gamma}'\right) - \left(\underset{\sim}{\Gamma} - \tfrac{1}{2}\underset{\sim}{\gamma}'\right)^2 \left(\underset{\sim}{\Gamma} - \tfrac{1}{2}\underset{\sim}{\gamma}'\right)$$

$$= \left(\cancel{\Gamma}^2 + \tfrac{1}{4}\gamma^2\right)\underset{\sim}{\Gamma} + \cancel{\left(\Gamma^2 + \tfrac{1}{4}\gamma^2\right)}\left(\tfrac{1}{2}\underset{\sim}{\gamma}\right) + \cancel{\left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}\right)}\underset{\sim}{\Gamma} + \left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}\right)\left(\tfrac{1}{2}\underset{\sim}{\gamma}\right)$$

$$+ \left(\cancel{\Gamma}^2 + \tfrac{1}{4}\gamma^2\right)\underset{\sim}{\Gamma} - \cancel{\left(\Gamma^2 + \tfrac{1}{4}\gamma^2\right)}\left(\tfrac{1}{2}\underset{\sim}{\gamma}\right) - \cancel{\left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}\right)}\underset{\sim}{\Gamma} + \left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}\right)\left(\tfrac{1}{2}\underset{\sim}{\gamma}\right)$$

$$- \left(\cancel{\Gamma}^2 + \tfrac{1}{4}\gamma'^2\right)\underset{\sim}{\Gamma} - \cancel{\left(\Gamma^2 + \tfrac{1}{4}\gamma'^2\right)}\left(\tfrac{1}{2}\underset{\sim}{\gamma}'\right) - \cancel{\left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'\right)}\underset{\sim}{\Gamma} - \left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'\right)\left(\tfrac{1}{2}\underset{\sim}{\gamma}'\right)$$

$$- \left(\cancel{\Gamma}^2 + \tfrac{1}{4}\gamma'^2\right)\underset{\sim}{\Gamma} + \cancel{\left(\Gamma^2 + \tfrac{1}{4}\gamma'^2\right)}\left(\tfrac{1}{2}\underset{\sim}{\gamma}'\right) + \cancel{\left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'\right)}\underset{\sim}{\Gamma} - \left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'\right)\left(\tfrac{1}{2}\underset{\sim}{\gamma}'\right)$$

$$= \underset{\sim}{\Gamma}\,\cancel{\left(\gamma^2/2 - \gamma'^2/2\right)} + \left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}\right)\underset{\sim}{\gamma} - \left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'\right)\underset{\sim}{\gamma}'$$

$$\left(\underset{\sim}{\Gamma} + \tfrac{1}{2}\underset{\sim}{\gamma}\right)\cdot\left\{\underset{\sim}{\Gamma}\,\cancel{\left(\gamma^2/2 - \gamma^2/2\right)} + \left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}\right)\underset{\sim}{\gamma} - \left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'\right)\underset{\sim}{\gamma}'\right\}$$

$$= \Gamma^2\cancel{\left(\gamma^2/2 - \gamma'^2/2\right)} + \tfrac{1}{4}\cancel{\left(\gamma^2 - \gamma'^2\right)}\left(\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma}\right) + \left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}\right)^2 + \tfrac{1}{2}\left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}\right)\gamma^2$$

$$- \left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'\right)^2 - \tfrac{1}{2}\left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'\right)\left(\underset{\sim}{\gamma}'\cdot\underset{\sim}{\gamma}\right)$$

$$= \cancel{\Gamma^2\left(\gamma^2 - \gamma'^2\right)/2} + \left(\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma}\right)\left[\overset{\checkmark}{\gamma^2/2} + \cancel{\gamma^2/4 - \gamma'^2/4}\right] + \overset{\checkmark}{\left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}\right)^2} - \left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'\right)^2$$

$$\overset{\checkmark}{-} \tfrac{1}{2}\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'\left(\underset{\sim}{\gamma}'\cdot\underset{\sim}{\gamma}\right)$$

$$= \left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}\right)^2 - \left(\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}'\right)^2 + \left(\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma}\right)\gamma^2/2 - \tfrac{1}{2}\left(\underset{\sim}{\gamma}'\cdot\underset{\sim}{\Gamma}\right)\left(\underset{\sim}{\gamma}'\cdot\underset{\sim}{\gamma}\right)$$

$$\cancel{+ \Gamma^2\left(\gamma^2 - \gamma'^2\right)/4 + \left(\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma}\right)\left(\gamma^2/4 - \gamma'^2/4\right)}$$

Now:

$$I_2 = \left(\frac{2kT}{m}\right)^{1/2} \iint d^3u\, d^3u_1 \int \sigma\, d\Omega\, \gamma\, f_0(u) f_0(u_1)\,(u^2 - 5/2)\,\underset{\sim}{u} \cdot \left[u^2 \underset{\sim}{u} + u_1^2 \underset{\sim}{u}_1 - u'^2 \underset{\sim}{u}' - u_1'^2 \underset{\sim}{u}_1'\right]$$

$$= \left(\frac{2kT}{m}\right)^{1/2} \iint \Gamma^2 d\Gamma\, d\omega_\Gamma \cdot \gamma^2 d\gamma\, d\omega_\gamma \int \sigma\Omega\, \gamma\, \frac{e^{-(2\Gamma^2+\frac{1}{2}\gamma^2)}}{\pi} \left[\cancelto{0}{\left(\Gamma^2 + \frac{\gamma^2}{4} - \frac{5}{2}\right)} + \underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma}\right] \left\{\cancelto{0}{(\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma})^2} - \cancelto{0}{(\underset{\sim}{\gamma}'\cdot\underset{\sim}{\Gamma})^2} + \frac{\gamma^2}{2}\left[\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma} - (\underset{\sim}{\gamma}'\cdot\underset{\sim}{\Gamma})\cos\chi\right]\right\}$$

$$= \left(\frac{2kT}{m}\right)^{1/2} \iint \Gamma^2 d\Gamma\, d\omega_\Gamma\, \gamma^2 d\gamma\, d\omega_\gamma \int \sigma\Omega\,\gamma\, \frac{e^{-(2\Gamma^2+\frac{1}{2}\gamma^2)}}{\pi} \left\{(\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma})\right\} \frac{\gamma^2}{2}\left[\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma} - (\underset{\sim}{\gamma}'\cdot\underset{\sim}{\Gamma})\cos\chi\right]$$

write $\underset{\sim}{\gamma}' = \cos\chi\,\underset{\sim}{\gamma} + \gamma\sin\chi\,\underset{\sim}{n}$

then $\underset{\sim}{\Gamma}\cdot\underset{\sim}{\gamma}' = \cos\chi\,\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma} + \gamma\sin\chi\,\underset{\sim}{n}\cdot\underset{\sim}{\Gamma}$

$(\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma})\gamma^2[\quad] = \frac{\gamma^2}{2}(\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma})^2 - \frac{\gamma^2}{2}\cos^2\chi(\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma})^2 - \frac{\gamma^3}{2}\sin\chi\cos\chi(\underset{\sim}{n}\cdot\underset{\sim}{\Gamma})(\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma})$

$(\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma})\gamma^2[\quad]$ ~~[illegible]~~

integral $\to 0$

$$= \frac{\gamma^2}{2}(\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma})^2(1-\cos^2\chi) - \frac{\gamma^2}{2}\gamma\sin\chi\cos\chi\,\cancel{(\underset{\sim}{n}\cdot\underset{\sim}{\Gamma})}(\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma})$$

$$I_2 = \left(\frac{2kT}{m}\right)^{1/2}\frac{2}{\pi^3}\iint(\Gamma^2 d\Gamma\, d\omega_\Gamma)(\gamma^2 d\gamma\, d\omega_\gamma)\int \gamma\sigma\, d\Omega\, e^{-(2\Gamma^2+\frac{1}{2}\gamma^2)}\,\frac{\gamma^2}{2}(1-\cos^2\chi)\underbrace{(\underset{\sim}{\gamma}\cdot\underset{\sim}{\Gamma})^2}_{\gamma^2\Gamma^2\cos^2\psi}$$

$$= \left(\frac{2kT}{m}\right)^{1/2}\frac{1}{2\pi^3}\iint \Gamma^2 d\Gamma\, d\omega_\Gamma \int d\omega_\gamma \int \sigma\, d\Omega\, e^{-(2\Gamma^2+\frac{1}{2}\gamma^2)}\underbrace{\cdot\gamma^2\cdot\gamma\cdot\underline{\gamma^2}\cdot\gamma^2\Gamma^2\cos^2\psi}\;\underbrace{(1-\cos^2\chi)}_{\sin^2\chi}$$

$$= \frac{1}{2\pi^3}\left(\frac{2kT}{n}\right)^{1/2}\int d\gamma\left(\gamma^7 4\pi e^{-\gamma^2/2}\right)\int\sigma\, d\Omega\sin^2\chi\int\left[4\pi\, d\Gamma\,\Gamma^4 \overset{\gamma^7\Gamma^2}{\exp(-2\Gamma^2)}\right]\underset{1}{\boxed{\int\frac{d\omega_\gamma}{4\pi}}}\int\frac{d\omega_\Gamma\cos^2\psi}{4\pi}$$

$$= \frac{1}{2\pi^3}\left(\frac{2kT}{m}\right)^{1/2}\int d\gamma\left(4\pi\gamma^7 e^{-\gamma^2/2}\right)\int\sigma(\gamma,\chi)\,d\Omega\sin^2\chi\int d\Gamma\left(4\pi\Gamma^4 e^{-2\Gamma^2}\right)\int\frac{d\omega_\Gamma\cos^2\psi}{4\pi}$$

$$d\omega_\Gamma = \sin\psi\, d\psi\, d\phi \Rightarrow \int \frac{\cos^2\psi \sin\psi\, d\psi\, d\phi}{4\pi} = \left[\frac{\cos^3\psi}{3}\right]_0^{\pi} \frac{(2\pi)}{4\pi} = \frac{1}{3}$$

$$\int_0^\infty x^{2n} e^{-x^2} dx = \frac{(2n-1)!!}{(2)^n 2}\sqrt{\pi} \qquad \int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n-1)!!}{2(2a)^n}\sqrt{\pi/a}$$

$$\int_0^\infty x^{2n+1} e^{-x^2} dx = \frac{n!}{2}$$

$n=2$
$a=2$

$$\int 4\pi \Gamma^4 e^{-2\Gamma^2} d\Gamma = \frac{4\pi 3}{2(4)^2}\sqrt{\pi/2} = \frac{3}{32}\sqrt{\frac{\pi}{2}}\cdot 4\pi = \frac{3}{8}\frac{\pi^{3/2}}{2^{1/2}}$$

So we have:

$$\int d\Gamma(\ )\cdot\int d\omega_\Gamma \cdot \int d\omega_\gamma = \frac{1}{8}\frac{\pi^{3/2}}{2^{1/2}}$$

$$\int d\Omega\, \sigma(\gamma,\chi)\sin^2\chi \equiv \sigma_{(2)}(\gamma)$$

hence

$$I_2 = \frac{1}{2\pi^3}\left(\frac{2kT}{m}\right)^{1/2}\int d\gamma\left(4\pi\gamma^7 e^{-\gamma^2/2}\right)\cdot\sigma_{(2)}\gamma\ \frac{\pi^{3/2}}{8\cdot 2^{1/2}}$$

$$= \frac{\pi^{3/2}}{16\pi^3\sqrt{2}}\left(\frac{2kT}{m}\right)^{1/2}\int d\gamma \overset{\sigma_{(2)}(\gamma)}{\wedge}\left(4\pi\gamma^7 e^{-\gamma^2/2}\right) = \frac{1}{16\pi^{3/2}}\left(\frac{kT}{m}\right)^{1/2}\int d\gamma\,\sigma_{(2)}(\gamma)\left(4\pi\gamma^7 e^{-\gamma^2/2}\right)$$

$$= \frac{4\pi}{16\pi^{3/2}}\left(\frac{kT}{m}\right)^{1/2}\int\left(\frac{2y\,dy}{\gamma}\right)\gamma^7 e^{-y^2}\sigma_{(2)}(y) = \frac{1}{4\pi^{1/2}}\left(\frac{kT}{m}\right)^{1/2}\int 2y\,dy\,(8y^6)e^{-y^2}\sigma_{(2)}(y)$$

$y^2 = \gamma^2/2$
$\gamma^2 = 2y^2$
$2\gamma d\gamma = 4y\,dy$
$d\gamma = \frac{2y\,dy}{\gamma}$
$\gamma^6 = 2^3 y^6$

$$\boxed{I_2 = 4\left(\frac{kT}{m\pi}\right)^{1/2}\int y^7 e^{-y^2}\sigma_{(2)}(y)} \qquad (33.31)$$

$$K = \left(\frac{2k^2T}{3m}\right)\frac{I_1^2}{I_2} = \left(\frac{2k^2T}{3m}\right)\frac{(15/4)^2}{4\left(\frac{kT}{m\pi}\right)^{1/2}\int y^7 e^{-y^2}\sigma_{(2)}(y)}$$

$$K_1 = \left(\frac{\cancel{8}k^2T}{\cancel{3}m}\right)\left(\frac{m\pi}{kT}\right)^{1/2}\left(\frac{15\cdot\overset{5}{\cancel{18}}\cdot\cancel{\text{[illegible]}}}{\underset{2}{\cancel{4}}\cdot\cancel{4}\cdot 4}\right)\left[\int y^7 e^{-y^2}\sigma_{(2)}(y)\,dy\right]^{-1}$$

$$= k\left(\frac{kT\pi}{m}\right)^{1/2}\cdot\frac{75}{32}\left\{\int y^7 e^{-y^2}\sigma_{(2)}(y)\,dy\right\}^{-1}$$

$$= \frac{75}{32}\left(\frac{\pi k^3 T}{m}\right)^{1/2}\left\{\int dy\, y^7 e^{-y^2}\sigma_{(2)}(y)\right\}^{-1} \qquad (33.32)$$

Rigid sphere:

$$\int dy\, y^7 e^{-y^2}\sigma_{(2)}(y) = \frac{2\pi d^2}{3}\int dy\, y^7 e^{-y^2}$$

$$= \frac{2\pi}{3}d^2\,\frac{3!}{2} = 2\pi d^2$$

$$K_1 = \frac{75}{32\,(2\pi d^2)}\left(\frac{\pi k^3 T}{m}\right)^{1/2} = \frac{75}{64 d^2}\left(\frac{k^3 T}{\pi m}\right)^{1/2}$$

$$\mu_1 = \frac{5}{8}\frac{(\pi m k T)^{1/2}}{2\pi d^2} = \frac{5}{16 d^2}\left(\frac{mkT}{\pi}\right)^{1/2}$$

$$\boxed{g^{-4/\alpha} = \cancel{2}\left(\frac{4kT}{m}\cancel{\gamma}\right)^{-2/\alpha} y^{-4/\alpha}}$$

$\cancel{\gamma = \sqrt{2}\,y}$

$\frac{\gamma^2}{2} = y^2$

Inverse power law:

$$\int dy\, y^7 e^{-y^2}\left\{2\pi\left(\frac{\alpha C_\alpha}{\tilde{m}}\right)^{2/\alpha} g^{-4/\alpha} A_2(\alpha)\right\} \qquad g = \left(\frac{2kT}{m}\right)^{1/2}\gamma = 2\left(\frac{kT}{m}\right)^{1/2} y$$

$$2\pi A_2(\alpha)\left(\frac{\alpha C_\alpha}{\tilde{m}}\right)^{2/\alpha}\int dy\, y^7 e^{-y^2}\cdot\cancel{2}\left(\frac{4kT}{m}\right)^{-2/\alpha} y^{-4/\alpha} = \cancel{4}\pi\left(\frac{4kT}{m}\right)^{-2/\alpha} A_2(\alpha)\left(\frac{\alpha C_\alpha}{\tilde{m}}\right)^{2/\alpha}\int dy\, y^7 y^{-4/\alpha} e^{-y^2}$$

$$= 4\pi A_2(\alpha)\left(\frac{4kT}{m}\right)^{-2/\alpha}\left(\frac{\alpha C_\alpha}{\tilde{m}}\right)^{2/\alpha}\int dy\, y^{(7-4/\alpha)} e^{-y^2} = 4\pi A_2(\alpha)\left(\frac{m}{4kT}\right)^{2/\alpha}\left(\frac{\alpha C_\alpha}{\tilde{m}}\right)^{2/\alpha}\int dy\, e^{-y^2} y^{(7-4/\alpha)}$$

$C_\alpha = e$

$\alpha = 1$

Inverse power law:

$$\int dy\, y^7 e^{-y^2}\left\{2\pi\left(\frac{\alpha C_\alpha}{\tilde{m}}\right)^{2/\alpha} g^{-4/\alpha} A_2(\alpha)\right\}$$

$$= 2\pi\left(\frac{\alpha C_\alpha}{\tilde{m}}\right)^{2/\alpha}\int dy\, y^7 e^{-y^2} g^{-4/\alpha} A_2(\alpha)$$

now $g = \left(\frac{2kT}{m}\right)^{1/2}\gamma$ and $\gamma = \sqrt{2}\,y$

hence

$$g = \left(\frac{4kT}{m}\right)^{1/2} y \Rightarrow g^{-4/\alpha} = \left(\frac{4kT}{m}\right)^{-2/\alpha} y^{-4/\alpha} = \left(\frac{m}{4kT}\right)^{2/\alpha} y^{-4/\alpha}$$

and

$$\int = 2\pi\left(\frac{\alpha C_\alpha}{m}\right)^{2/\alpha}\left(\frac{m}{4kT}\right)^{2/\alpha} A_2(\alpha)\int dy\, y^7 y^{-4/\alpha} e^{-y^2}$$

$$= 2\pi\left(\frac{\alpha C_\alpha}{4kT}\right)^{2/\alpha} A_2(\alpha)\int dy\, y^{(7-4/\alpha)} e^{-y^2}$$

But $\int dy\, y^{(7-4/\alpha)} e^{-y^2} = \frac{2^{2/\alpha}}{16}\Gamma(4-2/\alpha)$

hence

$$\int = \frac{\pi}{8}\left(\frac{\alpha C_\alpha}{2kT}\right)^{2/\alpha} A_2(\alpha)\,\Gamma(4-2/\alpha)$$

and

$$\mu_1 = \frac{5}{8}\left(\pi m k T^{1/2}\right) \Big/ \left\{\frac{\pi}{8}\left(\frac{\alpha C_\alpha}{2kT}\right)^{2/\alpha} A_2(\alpha)\,\Gamma(4-2/\alpha)\right.$$

$$= 5\left(\frac{mkT}{\pi}\right)^{1/2}\left(\frac{2kT}{\alpha C_\alpha}\right)^{2/\alpha}\left[A_2(\alpha)\,\Gamma(4-2/\alpha)\right]^{-1}$$

$7-4/\alpha+1 = 8-4/\alpha \qquad \frac{\nu}{2} = 4=2/\alpha$

$$= 4\pi A_2(\alpha)\left(\frac{\alpha C_\alpha}{4kT}\right)^{2/\alpha}\int dy\, e^{-y^2} y^{(7-4/\alpha)}$$

$$\int_0^\infty x^{\nu-1} e^{-\beta x^2-\gamma x}\,dx = (2\beta)^{-\nu/2}\Gamma(\nu)\exp(\gamma^2/8\beta)D_{-\nu}(\gamma/\sqrt{2\beta})$$

$$= (1\cdot 2)^{-(4-2/\alpha)}\Gamma(4-2/\alpha) = \frac{2^{2/\alpha}}{2^4} = \frac{2^{2/\alpha}}{16}\Gamma(4-2/\alpha)$$

Thus

$$\int = 4\pi A_2(\alpha)\left(\frac{\alpha C_\alpha}{4kT}\right)^{2/\alpha}\cdot\frac{2^{2/\alpha}}{16}\Gamma(4-2/\alpha)$$

$$= \frac{\cancel{4}\pi A_2(\alpha)}{\cancel{16}\,4}\left(\frac{\alpha C_\alpha}{2kT}\right)^{2/\alpha}\Gamma(4-2/\alpha)$$

$$= \frac{\pi A_2(\alpha)}{4}\left(\frac{\alpha C_\alpha}{2kT}\right)^{2/\alpha}\Gamma(4-2/\alpha)$$

$$\mu = \frac{5}{8}(\pi mkT)^{1/2}\Big/\left\{\frac{\pi A_2(\alpha)}{4}\left(\frac{\alpha C_\alpha}{2kT}\right)^{2/\alpha}\Gamma(4-2/\alpha)\right\}$$

$$= \frac{5}{\cancel{8}\,2}(\pi mkT)^{1/2}\cdot\frac{\cancel{4}}{\pi A_2(\alpha)}\left(\frac{2kT}{\alpha C_\alpha}\right)^{2/\alpha}\Big/\Gamma(4-2/\alpha)$$

$$= \frac{5}{2}\left(\frac{mkT}{\pi}\right)^{1/2}\left(\frac{2kT}{\alpha C_\alpha}\right)^{2/\alpha}\Big/\left[A_2(\alpha)\Gamma(4-2/\alpha)\right]$$