TITLE: IMPROVED CALCULATION OF THE PROMPT FISSION NEUTRON SPECTRUM FROM THE SPONTANEOUS FISSION OF $^{252}\text{Cf}$

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An improved calculation is presented for the prompt fission neutron spectrum \( N(E) \) from the spontaneous fission of \(^{252}\text{Cf}\). In this calculation the fission-spectrum model of Madland and Nix is used, but with several improvements leading to a physically more accurate representation of the spectrum. Specifically, the contributions to \( N(E) \) from the entire fission-fragment mass and charge distributions will be calculated instead of calculating on the basis of a seven-point approximation to the peaks of these distributions as has been done in the past.

Therefore, values of the energy release in fission, fission-fragment kinetic energy, nuclear level density, and compound nucleus cross section for the inverse process will be considered on a point-by-point basis over the fragment yield distributions instead of considering averages of these quantities over the peaks of the distributions. Particular attention will be given to the energy-dependent compound nucleus cross sections and to the nuclear level density model. Other refinements to the calculation of \( N(E) \) will also be discussed.

Results will be presented and compared with earlier calculations of the spectrum and with recent experimental measurements of the spectrum.
Summary of Theoretical Model

1. Based on standard nuclear evaporation theory

2. Accounts for effects of:
   - motion of fission fragments
   - distribution of fission-fragment excitation energy
   - energy dependence of cross section for inverse process of compound nucleus formation
   - multiple-chance fission effects at high incident neutron energy
3. Original implementation of model:

- **motion of fission fragments:**

  calculate *average* kinetic energy per nucleon of the *average* light fragment and likewise for the *average* heavy fragment

  \[ E_{f}^{L} = \left( \frac{A_{H}}{A_{L}} \right) \left( \frac{<E_{f}^{\text{tot}}>/A}{A} \right), \text{ and} \]

  \[ E_{f}^{H} = \left( \frac{A_{L}}{A_{H}} \right) \left( \frac{<E_{f}^{\text{tot}}>/A}{A} \right), \text{ where} \]

  \[ A_{L}, A_{H}, \text{ and } <E_{f}^{\text{tot}}> \text{ are all *average* quantities} \]

  and \( A \) is mass number of fissioning nucleus.
distribution of fission-fragment excitation energy:

\[
P(T) = \begin{cases} 
2T/T_m^2 & \text{if } T \leq T_m \\
0 & \text{if } T > T_m 
\end{cases}
\]

\[
<E^*> = <E_r> + E_n + B_n - <E_{f}^{tot}>
\]

\[
= a T_m^2, \text{ where}
\]

\(<E^*> \) is the total average fission-fragment excitation energy,

\(<E_r> \) is the total average energy release in fission,

\(E_n, B_n \) are the kinetic and separation energies of the neutron inducing fission,

\(<E_{f}^{tot}> \) is the total average fission-fragment kinetic energy, and

\(a \) is the nuclear level-density parameter.
The accuracy of \( T_m \) therefore depends upon the accuracy of the difference between two large similar numbers, \(<E_r>\) and \(<E_r^{\text{tot}}>\), and upon the accuracy of the nuclear level-density parameter \( a \).

We used a seven-point approximation for \(<E_r>\), experimental values for \(<E_r^{\text{tot}}>\), and a Fermi gas level-density parameter \( a = A/(\text{const}) \).
• **energy dependence of cross section for inverse process of compound nucleus formation:**

\[ \sigma_c(\varepsilon) \] calculated on a 100 point grid, to 40 MeV, using the Becchetti-Greenlees potential

OMP studies showed that the *shape* of \( \sigma_c(\varepsilon) \) vs \( \varepsilon \) is the dominant effect

• **multiple-chance fission effects at high incident energy:**

fission probabilities, \( P_f \), from Back and Britt experiments

neutrons emitted prior to fission described by evaporation spectra, including energy-dependent cross sections for the inverse process, \( \phi(E,\sigma_c) \)

distribution of excitation energy in fissioning nucleus, \( D(E^*,\sigma_c) \), given by *complement* of evaporation spectrum \( \phi(E,\sigma_c) \)
• basic spectrum in original implementation of model:

\[ N(E) = \frac{1}{2} (N(E, E^L_r, \sigma^L_\omega) + N(E, E^H_r, \sigma^H_\omega)) \]
Fig. 1. Energy spectrum for $^{252}$Cf(sf) in laboratory conditions.
Fig. 2.
SPONTANEOUS FISSION OF $^{252}$Cf

refined implementation of model

a) include contributions to $N(E)$ from *entire* fission-fragment mass distribution $Y(A)$ instead of the *seven-point approximation*.

b) include contributions to $N(E)$ from the fission-fragment charge distribution $P(Z)$ for each value of $A$.

c) include contributions to $N(E)$ from the inverse process of compound nucleus formation for the *entire* fission-fragment mass and charge distributions, instead of for the peaks of these distributions.

d) include explicit gamma-ray de-excitation of fission fragments.

e) include effects of center-of-mass anisotropy.

f) investigate scission neutron question.

We have addressed (a), (b), (c), and (f) in the present work.
basic spectrum in refined model

\[ N(E) = \sum_{A} \frac{\nu(A)}{\nu_{\text{tot}}} Y(A) \sum_{Z} P(Z) N(E,E_f(A),\sigma_c(Z,A),T_m(Z,A)) \]

where \( \bar{\nu}_{\text{tot}} = \sum_{A} \bar{\nu}(A) Y(A) \) is the total average multiplicity, \( Y(A) \) is the fission-fragment mass distribution, \( P(Z) \) is the fission-fragment charge distribution, and \( A \) (or \( A_L \) and \( A_H \)) and \( Z \) are no longer average quantities.
fission-fragment charge distribution $P(Z)$

$$P(Z) = \left(\frac{1}{\sqrt{\pi c}}\right) \exp\left(-\frac{(Z-Z_p)^2}{c}\right),$$

which is normalized by $P_{\text{tot}} = \sum Z P(Z)$, and where the sum over $Z$ is for all contributing fragment yields for a given fragment mass number $A$.

the most probable charge, $Z_p$, is obtained using a corrected unchanged charge distribution (UCD) assumption, due to Unik et al.,

$$\frac{(Z_p^L - \frac{1}{2})}{A_L} = \frac{(Z_p^H + \frac{1}{2})}{A_H} = \frac{Z_c}{A_c},$$

and

$$c = 2(\sigma^2 + 1/12), \quad \sigma = 0.40 \ (\text{Reisdorf et al.})$$
- \( T_m(Z,A) \)

for spontaneous fission,

\[
T_m = \left\{ \left( \frac{E_f(Z,A) - E_{f_{\text{tot}}(Z,A)}}{a(A_c)} \right) \right\}^{1/2}
\]

where

\[
E_f(Z,A) = M - M_l(Z_l,A_l) - M_H(Z_H,A_H)
\]

is calculated using experimental masses where they exist and otherwise the calculated masses of Möller and Nix (1986),

and experimental values of \( E_{f_{\text{tot}}} \), averaged over \( Z \), are taken form the measurements of Schmitt et al. (1966),

and the level-density parameter \( a(A_c) \) in the Fermi gas model is still given by \( a = A_c/\text{const} \).

- kinetic energy per nucleon of fission fragments, \( E_f(A) \)

calculated using experimental values of \( E_{f_{\text{tot}}}(Z,A) \) due to Schmitt et al. (1966) and momentum conservation.

- inverse cross section \( \sigma_c(Z,A) \)

calculated using the Becchetti-Greenlees potential for each fission fragment.
• fission-fragment mass distribution $Y(A)$
  
  experimental values used from the measurements of Schmitt et al. (1966)

• $\bar{v}(A)$
  
  experimental values used from the measurements of Walsh and Boldeman (1977).
Fig. 4.

Heavy Fragment Mass Number $A_H$

Total Fragment Kinetic Energy $E_{kH}(A_H)$ (MeV)

Heavy Fragment Yield $Y(A_H)$ (%)
results

We represented the fission-fragment mass and charge distributions using 28 fragments:

- 14 approximately equispaced $A$ values ranging from 88 to 164
- 2 values of $Z$ per $A$ value, the nearest integer values to $Z_p$

We used

- 28 optical-model calculations of $\sigma_c(\epsilon)$
- 28 calculations of $E_r$
- 14 calculations of $E_r(A)$
- 14 experimental values of $E^{\text{tot}}_r(A)$
- 14 experimental values of $\bar{\nu}(A)$
Fig. 1.
Fig. 7.
- We found that the refined model calculation compared to the original model calculation is
  - harder at the high energy end,
  - harder at the low energy end, and
  - softer in the intermediate region.

- We found that the agreement with the experimental spectrum of Poenitz and Tamura (1982) is improved with the refined model calculation, for an identical value of the level-density parameter, namely, $a = A/9.15$

- Using this same value of the level-density parameter, we found unsatisfactory agreement with the evaluated spectrum of Mannhart (1987).

- Performing a least-squares adjustment to the evaluated spectrum of Mannhart (1987), we determined a level-density parameter $a = A/9.40$. We again found unsatisfactory agreement with the evaluated spectrum of Mannhart (1987).
We conclude that

- the fission-fragment mass and charge distribution should be represented more completely - more than 28 fragments,

- the effects of center-of-mass anisotropy should be included, and

- explicit accounting of gamma-ray de-excitation effects should be included.
Fig. 1  Ratio of the previous least-squares adjusted Los Alamos spectrum and the experimental spectrum of Poenitz and Tamura (1982) to the least-squares adjusted Maxwellian spectrum, for $^{252}\text{Cf}(sf)$.

Fig. 2  Ratio of calculated to experimental integral cross sections for the spontaneous fission of $^{252}\text{Cf}$ as a function of the effective neutron threshold energy for the reaction. The calculated values are obtained using the previous least-squares adjusted Los Alamos spectrum together with ENDF/B-V pointwise cross sections. The experimental values are those of Grundl et al. and Kobayashi et al.

Fig. 3  Similar to Fig. 2 except that the calculated integral cross sections are obtained using the least-squares adjusted Maxwellian spectrum.

Fig. 4  Experimental values of the total fission-fragment kinetic energy for $^{252}\text{Cf}(sf)$ as a function of the heavy fragment mass number, determined by Schmitt et al. (1966).

Fig. 5  Experimental values of the heavy fragment yield for $^{252}\text{Cf}(sf)$ as a function of the heavy fragment mass number, determined by Schmitt et al. (1966).
Fig. 6  Comparison of the previous least-squares adjusted Los Alamos spectrum, based on considerations of the peaks of the fission-fragment mass and charge distributions, and the present Los Alamos spectrum, based on considerations of the entire fission-fragment mass and charge distributions. The level-density parameter in both calculations is given by $a=A/(9.15 \text{ MeV})$.

Fig. 7  Ratio of the present Los Alamos spectrum and the evaluated spectrum of Mannhart (1987) to a Maxwellian spectrum with $T=1.42 \text{ MeV}$, for $^{252}\text{Cf}(sf)$. The nuclear level-density parameter is given by $a=A/(9.15 \text{ MeV})$.

Fig. 8  Ratio of the present least-squares adjusted Los Alamos spectrum and the evaluated spectrum of Mannhart (1987) to a Maxwellian spectrum with $T=1.42 \text{ MeV}$, for $^{252}\text{Cf}(sf)$. The adjusted nuclear level-density parameter is given by $a=A/(9.40 \text{ MeV})$. 