ELECTROMAGNETIC OBSERVATION OF IMPLOSION AS AFFECTED BY THE H.E. PRODUCT GAS

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ABSTRACT

It is estimated that the product gases of the H.E. layer surrounding an imploded sphere are sufficiently transparent to slow magnetic signals to permit the magnetic observation of the implosion, but that, through their dielectric properties, they alter the lowest electric mode of vibration enough to make the radio resonance method of observation impracticable.

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Two quite different methods have been proposed for implosion observations which might be impeded by the conductivity and dielectric properties of the ionized gas formed of the explosion products. They are the magnetic method proposed by Fowler and by Woodward and explored experimentally by Fowler, and the radio resonance method proposed more recently by Alvarez and by Manley. In the former, a magnetic field collapses with the implosion, and its success depends on sufficient penetration of the gas by a magnetic signal of frequencies around 10^6 sec^-1. In the latter, the size of the sphere is indicated by resonance with a radio wave of frequency around 10^{10} sec^-1, and it is necessary that the fundamental frequency of the conducting sphere be changed by not more than a few percent by the surrounding gases, and the resonance not broaden excessively.

This document contains 7 pages.
Here we attempt to make an estimate of the conductivity of the hot H.E. products during the implosion, although such an estimate must necessarily be quite crude because of the uncertain composition and temperature of the gas at the times of interest. The result is that the conductivity is not great enough to impair the observations unless polyatomic carbon is present. The estimated dielectric constant is, however, great enough to make any interpretation of the resonances depend on the uncertain outline of the explosive gases, so that method is considered impracticable.

**CONDUCTIVITY OF PRODUCT GAS**

The molecules probably present in the H.E. product, and their ionization potentials $E$, are listed in Table I. The numbers present per molecule of TNT and of tetryl, each according to two different formulas, are listed as taken from an early report of Kistiakowsky and Wilson, in order to give a rough idea of the probable abundance in a somewhat arbitrary mixture.

**TABLE I. Composition of explosion product gas.**

<table>
<thead>
<tr>
<th>Molecule</th>
<th>$E$/ev.</th>
<th>Number per molecule of TNT</th>
<th>Number per molecule of Tetryl</th>
<th>Contribution to $(e-1)$ for each prod. molec. per H.E. molec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (atom)</td>
<td>11.2</td>
<td>1</td>
<td>3-1/2</td>
<td>.04</td>
</tr>
<tr>
<td>$O_2$</td>
<td>12.5</td>
<td></td>
<td></td>
<td>.11</td>
</tr>
<tr>
<td>$H_2O$</td>
<td>12.7</td>
<td>2-1/2</td>
<td>1</td>
<td>.053</td>
</tr>
<tr>
<td>CO</td>
<td>14.1</td>
<td>6</td>
<td>3-1/2</td>
<td>.021</td>
</tr>
<tr>
<td>$H_2$</td>
<td>15.37</td>
<td>2-1/2</td>
<td>1-1/2</td>
<td>.045</td>
</tr>
<tr>
<td>$N_2$</td>
<td>15.51</td>
<td>1-1/2</td>
<td>1-1/2</td>
<td>.045</td>
</tr>
</tbody>
</table>
A rough idea of the abundance will here suffice because the number of conduction electrons is a very sensitive function of temperature and lowest $E$, but a relatively insensitive function of the abundance of molecules having that lowest $E$, and practically independent of the other molecules, since the next lowest $E$ is several $kT$ above the lowest. The number of electrons (and we consider only electronic conduction) is determined by the Law of Mass Action, written for the case in which the number of ions is equal to the number of electrons, $n$, per cc:

$$n^2/N = (2w_1/w_m)(2\pi mkT/h^2)^{3/2} e^{-E/kT}$$

(1)

Here $N$ is the density of molecules of lowest $E$, the $w$'s are statistical weights of the populated states of ion and molecule, and $m$ is electron mass. In evaluating $N$, we assume that it refers to C atoms, that there are say two of them per explosive molecule, and that the gas has about the density of the original explosive, 1.5 g/cc, at. wt. 220. For C, we may, with greater refinement than necessary, put $2w_1/w_m = 4/3$, taking into account the ground states $2p^1s$ and $3p^1s$.

The electron mobility at atmospheric pressure is $12000 \text{ cm}^2 \text{ volt}^{-1} \text{ sec}^{-1}$ for the most abundant molecule CO, and 18000 for $N_2$ (Wahlin, Phys. Rev., 35, 1570 (1930)), at low fields, and drops off gradually for higher fields. In our composite gas we will be content with the rough value obtained by attributing to each molecule the low field cross section of CO, since experimental values are not at hand for all of the molecules. We thus obtain a conductivity

$$\sigma = 1.4 \times 10^{10} T^{3/2} e^{-E/2kT} \text{ (c.g.s.)}$$

(2)

As a safe upper limit, we may take the temperature of the explosive to be
T = 4500° C, or 2kT = (6/6) ev, which, with E = 11.2 ev, makes \( \sigma = 10^7 \) c.g.s.e.s.u.

**DIELECTRIC CONSTANT OF THE GAS**

At the frequencies contemplated, considerably below rotational frequencies of a molecule, the dielectric constant \( \varepsilon \) exceeds unity by an amount consisting of two contributions, one arising from the permanent electric dipole moments of the molecules and inversely proportional to the temperature, another due to induced polarization and independent of temperature. The first is about 1.4 times the second for H$_2$O at 4000° C, while for CO at this temperature the first is about 0.3 percent of the second and in the other molecules listed in Table I the first contribution is absent. Thus, aside from water, the contributions of the various molecules to \( (\varepsilon - 1) \) are obtained from measured values (Zahn, Phys. Rev., 24, 400 (1924) and 32, 497 (1928)) merely by multiplying by the ratio of densities. If we assume the explosive to have a density of about 1.5 and a molecular weight of about 220, we find the contributions to \( (\varepsilon - 1) \) listed in the last column of Table I, for each gas molecule per molecule of the original explosive. If we take for example the product gas at its original density and with the average of the compositions listed in Table I, we find \( (\varepsilon - 1) = 0.6 \). As the gas expands, this is reduced proportional to the density, with a slight upward correction for the reduced temperature. There might also be changes in composition and other complications.

**SIGNAL ATTENUATION IN THE CONDUCTING LAYER**

A plane wave in a conducting medium dies off according to the real-exponent part of a space factor \( e^{ikx} \) where \( k = \left[ \frac{\mu \omega (\varepsilon \omega - 4\pi i \sigma)}{c} \right]^{1/2}/c \). For small values of \( \sigma \) this leads to an attenuation factor \( e^{-2\sqrt{\mu/\varepsilon} k_x/c} \). Insertion
of $\sigma = 10^7 \text{sec}^{-1}$, with $\mu$ and $\varepsilon$ unity, gives an attenuation distance of five meters, indicating that no trouble is to be anticipated from this source at high frequencies. With $\omega \ll \sigma$, the attenuation factor is $e^{-\left(2\pi\mu\omega\right)^{1/2}x/c}$ and the attenuation distance about 150 meters for a frequency of $10^6 \text{sec}^{-1}$, again quite innocuous.

OSCILLATIONS OF A CONDUCTING SPHERE WITHIN A CONCENTRIC DIELECTRIC SPHERE

The fundamental electric mode of oscillation, which is presumably the one that would be observed, within a conducting sphere may be represented in terms of the Bessel function of order $3/2$ (and derivatives thereof — see Born, Optik, p. 280; Stratton, Electromagnetic Theory, p. 567), while the oscillations to which it joins in the intermediate dielectric sphere are represented by a sum of Bessel and Hankel functions, and in the outer infinite space by a Hankel function only, all of order $3/2$. If we call the coefficients of these four functions $A$, $B$, $C$, and $D$, respectively, the boundary conditions $E_{\text{tang}}$ continuous and $H_{\text{tang}}$ continuous at the inner boundary $r = a$ and at the outer boundary $r = b$ may be written

$$
\left(\frac{k_1}{\mu_1}\right)A_{1a} = \left(\frac{k_2}{\mu_2}\right)(B_{j2a} + C_{h2a})
$$

$$
\left(\frac{1}{k_1}\right)A_{[p]_{1a}} = \left(\frac{1}{k_2}\right)(B_{[p]_{2a}} + C_{[h]_{2a}})
$$

$$
\left(\frac{k_2}{\mu_2}\right)(B_{j2b} + C_{h2b}) = \left(\frac{k_3}{\mu_3}\right)D_{3b}
$$

$$
\left(\frac{1}{k_2}\right)(B_{[p]_{2b}} + C_{[h]_{2b}}) = \left(\frac{1}{k_3}\right)D_{[h]_{3b}}
$$

Here $j_{2a} = j_{2a}$, or $j_{1}(\varphi_{2a})$ in Stratton's notation, with $\varphi_{2a} = k_2a$. The subscripts 1, 2, 3 refer to the metal, dielectric, and outer space, respectively, and in each medium we have $k$ defined as above. The primes mean differentiation with respect to $r$, and
\[ j(z) = z^{-2} \sin z - z^{-1} \cos z \]
\[ h(z) = -e^{iz}(1 + iz)/z. \]

For the high frequencies here of interest, the metal sphere affects the outside world only through a skin effect independent of further details about \( k_1 \). We consider \( k_1 \) much larger than unity and much larger than \( k_2 \) and \( k_3 \) and have as the condition for the existence of solutions B, C, and D the vanishing of the determinant formed by their coefficients in the last three rows and columns of (3). In developing this condition further, we note that the change in \( k_2 \) due to conductivity is very small, consider for simplicity a small change \( \Delta \varepsilon \) in \( \varepsilon_2 \) from unity, put all \( \mu \) equal to unity, and find the following formula for the consequent change \( \Delta \varepsilon \) in \( \varepsilon_1 \):

\[ \frac{\Delta \varepsilon}{\varepsilon} = (1/2) \left\{ \left( \left[ \frac{\partial^2}{\partial \varepsilon_1^{\mu} \partial \varepsilon_2^{\mu}} \right] \right) \left( 1 + \frac{2i}{\sigma} - \frac{4\varepsilon_2^{\mu}}{\sigma^2} - \frac{2i}{\sigma^3} \right) e^{2i\sigma} - 1 \right\} \Delta \varepsilon \quad (5) \]

Here \( \rho_0 = \rho_0/\sigma = (3^{1/2} + i)/2 \) is the solution of the unperturbed problem \( [\rho]_0 = 0 \), to which our problem reduces when \( k_2 = k_3 = \sigma/\rho_0 \) as is most easily seen by developing the three-row determinant in terms of minors of the first row. \( \sigma \) is a corresponding quantity for the outer surface, \( \sigma = (b/a)\rho_0 \). In evaluating this expression we first take \( (b/a) = 7/4 \), which corresponds to a gas occupying the volume originally occupied by the explosive with a charge-mass ratio of about 4, and then take \( (b/a) = 2 \), which corresponds to the same gas after a 50 percent expansion such as might take place during the observation. In the former case we might expect \( \Delta \varepsilon = 0.6 \) and in the latter 0.4. The results of the evaluation for these cases are shown in Table II. It must be emphasized that these are the results of a series expansion to the first order term in \( \Delta \varepsilon \) only, and \( \Delta \varepsilon \) is not very small compared to unity, so they are to be used only
for making a preliminary judgment of the probable practicability of the method, and not for evaluating in detail results obtained by the method. The latter could presumably be done better by means of calibration shots of explosive layers surrounding solid metal spheres.

One sees not only that the effect of the gas on the resonant frequency is large, but that the uncertainty in this effect due to uncertainty in the state of expansion of the gas is of the order of twenty percent, enough to make the method impracticable. The breadth of tuning is determined essentially by the square of the ratio of the imaginary part of \( \omega \) to the real part, and is on the verge of being too large for a bare conducting sphere (the variation of the steady amplitude \( A(\alpha) \) near resonance is given by \( A^2 A'' A' = -\frac{9}{2} \)). The imaginary part is decreased a little less than the real part, so the tuning is also made somewhat broader by the gas.