Criticality of the Water Boiler, Number of Delayed Neutrons, and Dispersion of the Neutron Emission Per Fission

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CRITICALITY OF THE WATER BOILER, NUMBER OF DELAYED NEUTRONS,
AND DISPERSION OF THE NEUTRON EMISSION PER FISSION

Upon re-examination of LA-163 in the course of preparatory work for the
Technical Series it was discovered that there were errors in this report.

Table I turned out to contain faulty values for the composition of the
solutions which strongly influenced our results. Table I is hereby appended with
all corrections made.

p. 26: The last paragraph should read:

Using these cross sections one obtains the following results:

<table>
<thead>
<tr>
<th>Scattering Cross Section per cc</th>
<th>Absorption Cross Section per cc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mock Solution</td>
<td>2.749 cm²</td>
</tr>
<tr>
<td>Normal 25 Solution</td>
<td>2.734 cm²</td>
</tr>
<tr>
<td>Ratio</td>
<td>1.005</td>
</tr>
</tbody>
</table>

p. 27: This page should read:

We see that the scattering cross section is almost the same, and any af-
facts due to it may be neglected. As far as absorption goes, our bubble was more
effective than it should have been, consequently it gave higher values of $\Delta M$.

$$\Delta M_{\text{corrected}} = \frac{1.98}{1.075} = 1.84 \text{ gms of } 25$$

The $\Delta K$ by Eq. (14) is

$$\Delta K = \frac{\text{Volume of bubble}}{\text{Volume of boiler}} = \frac{15.17 \text{ cm}^3}{15 \times 10^3 \text{ cm}^3} = 1.01 \times 10^{-3}$$

with the same uncertainty in the volume of the boiler. Thus we get that:

1 gm of 25 is equivalent to $1.01 \times 10^{-3}/1.77$ units of $\Delta K$

1 gm of 25 = $5.48 \times 10^{-4} \Delta K$
i.e., calling the conversion factor $c_1$:

$$c_1 = 5.48 \times 10^{-4} \Delta K/gm$$

B. The $\gamma_f$ and $\bar{T}_p$ Experiment - same for pages, 27, 28, and 29.

The $\bar{T}_p$ on the other hand can be obtained from the 884-rpm curve since the phase lag is large giving

$$\bar{T}_p \sim 135 \pm 20 \text{ microseconds}$$

The last line should read:

i.e., if $c_1 = 0.000548 \Delta K/gm$, $\gamma_f = 0.00855$

p. 31 and 32: These pages should read:

V. Discussion of Results:

The value of $c_1$ should be fairly good and applicable to the case of the $\gamma_f$ and $\bar{T}_p$ experiment since the concentration of 25 in the boiler was identical. Thus no change in $c_1$ due to that effect is expected. The cross sections used are known to within a few percent and better.

In the $\gamma_f$ and $\bar{T}_p$ experiment it can be seen that large errors must be expected. If, however, we take the value of $\gamma_f = 0.00855$ at face value we may draw the following conclusions:

From LA-101 we see that

$$\bar{V}_p^2 - \bar{V}_p = Y(\bar{V}_p)^2 (\gamma_f)^2 / \epsilon$$

$Y$ was found to be 4.17. A preliminary value of $\epsilon = 3.60 \times 10^{-4}$ was given in LA-101. Since then a better value of the efficiency of the 25 chamber has been obtained giving

11) See LA-101, page 11 where $\epsilon = 2.51 \times 10^{-7}$ counts/fission was given. The value should be $2.42 \times 10^{-7}$ counts/fission.
the BF$_3$ chamber efficiency as $3.51 \times 10^{-4}$. If we take $\bar{v}$ to be 2.47 then 
\[ (\bar{v}^2) = 6.10 (1-f)^2. \]

Hence
\[ \bar{v}^2 - \bar{v} = 5.30 (1-f)^2 \]

This brings us to the question regarding the value of $f$. Chicago measurements vary 
from 0.006 to 0.008 giving $\bar{v}^2 - \bar{v}$ varying from 5.22 to 5.24. This shows that $\bar{v}^2 - \bar{v}$ 
does not depend sensitively on $f$ and thus we may take 
\[ \bar{v}^2 - \bar{v} = 5.2 \]

In report LA-471 an attempt has been made to calculate $\gamma$ in first approximation only which yielded a value of $\gamma = 1.65$ as an upper limit, using this value 
for $\gamma$, $f = 0.052$. It should be remembered that the theoretical calculation of $\gamma$ is 
very crude and this value of $f$ is therefore in reasonably good agreement.

It would be a mistake to infer anything very definite regarding the actual number of neutrons emitted from each fragment.

It is also not fair to deduce anything regarding the question of immediate 
versus evaporated emission of neutrons on fission. It can be shown that if one as-
sumes that neutrons evaporate from each fragment, i.e., 1.25 neutrons per fragment 
on the average, we get values of $\bar{v}^2 - \bar{v}$ very close to those expected from direct 
splitting.

The value of $\bar{v}^2 - \bar{v}$ should thus be used only as an entity in itself for such 
calculations as the probability of predetonation where it is needed.

The value of $\gamma = 135 \pm 20$ microseconds is interesting as a differential 
quantity of the particular water boiler since it confirms theoretical calculation 
as to its order of magnitude.
ADDITIONAL REMARKS

The value of $\gamma f$ expressed in grams, i.e., 15.6 grams is unaffected by our change in $c_1$. The value of $\gamma f$ in grams does not depend on $c_1$ at all in the case of the direct-analysis method. In the case of the reproduction method (Case II) only the ratio of $\gamma p/\gamma f$ matters, which is very insensitive to a change in $c_1$. This can be seen from Eq. (45) and (46). We note from Eq. (45) that $\gamma f \sim c_1$ and from (46) that $\gamma p \sim c_1(1 - f)$ thus

$$\frac{\gamma p}{\gamma f} \sim 1/(1 - f)$$

which is not very sensitive to $c_1$. Thus our value of $\gamma f$ in grams is unaffected.

$\gamma p$ itself is essentially proportional to $c_1$ when evaluated from the direct analysis method. This method yields $\gamma p = 132$ microseconds from the 884-rpm data by use of Eq. (46) as a first approximation. The dashed curve of Fig. 8 is the experimental curve as originally shown in Fig. 7 where the limits of error are indicated. Fig. 8 also shows the theoretical points calculated by the use of the reproduction method Case I, i.e., Eq. (28). For the calculation of the theoretical points the values of $c_1 = 5.48 \times 10^{-4}$ and $\gamma f = 0.0085$ were used and five values of $\gamma p$ tried, namely $\gamma p = 90, 120, 132, 140$ and 150 microseconds. Curves are actually drawn for only two sets of points. A comparison with Fig. 7 shows that the experimental phase shift is best reproduced by a $\gamma p \sim 135 \pm 20$ microseconds. It can be seen that the error in $\gamma p$ is probably of the order 20 microseconds as indicated.
### TABLE I

#### A. Boron-Bubble Experiment

**Composition of Mock Solution and of Normal 25 Solution at 39° C**

<table>
<thead>
<tr>
<th>Element</th>
<th>No. of gm/cm² in normal 25 solution</th>
<th>No. of gm/cm² in mock solution</th>
<th>Absorption cross section used</th>
<th>Scattering cross section used</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.03878</td>
<td>0.0000927</td>
<td>6.4(12)</td>
<td>---</td>
</tr>
<tr>
<td>28</td>
<td>0.2256</td>
<td>0.2247</td>
<td>12.11</td>
<td>8.2</td>
</tr>
<tr>
<td>B</td>
<td>none</td>
<td>0.001749</td>
<td>72(13)</td>
<td>---</td>
</tr>
<tr>
<td>H</td>
<td>0.105</td>
<td>0.1059</td>
<td>0.3</td>
<td>41</td>
</tr>
<tr>
<td>O</td>
<td>0.942</td>
<td>0.940</td>
<td>0.0016</td>
<td>4.2</td>
</tr>
<tr>
<td>S</td>
<td>0.0357</td>
<td>0.0303</td>
<td>0.47</td>
<td>1.5</td>
</tr>
</tbody>
</table>

12) See LA-140.

13) Effective cross section including effect of high-energy neutrons, page 26.

14) " " " " " " " " " " " " as calculated by E. Fermi.