LIGHTWEIGHT NUCLEAR ROCKET REACTORS

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LIGHTWEIGHT NUCLEAR ROCKET REACTORS

(Title Unclassified)

by

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Appendix B

by

William Kirk

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NUCLEAR ROCKET AND RAM-JET ENGINES
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ABSTRACT

Up to the present, the advantage of high specific impulse offered by nuclear heat exchanger rocket engines utilizing hydrogen as a propellant has been partially offset by the reactor weight, since a large system is required for full effectiveness. The presently contemplated minimum weight $^{235}U$-graphite (Kiwi) reactor will weigh approximately 8000 pounds and deliver 50,000 pounds of thrust (1000 Mw power).

It is our belief that, applying the concept of fast reactors utilizing $^{235}U$ carbide ($^{235}U$-ZrC solid solutions) or cermet ($^{235}U_2O_2$-W) fuel elements, one is capable of producing reactors weighing 1200 to 3000 pounds for the 1000 Mw power level (depending upon the fuel element material) and of reducing the minimum weight of nuclear engines to 900 to 2000 pounds for a 200 Mw (10,000 pounds thrust) level, including pressure shell plus 100 pounds of miscellaneous extras.

Secondly, we show that the use of $^{233}U$ in place of $^{235}U$ can reduce the weights of the above systems by a factor of 2 in the 1000 Mw range and a factor of 2.5 at the 200 Mw level. This includes halving the minimum weight of graphite reactors. Alternatively, one can ultimately use the superior neutronic properties of $^{233}U$ to increase the exit gas temperature (and thus specific impulse) in carbide reactors by using carbide solid solutions containing smaller percentages of UC$. Problems associated with the use of fast reactors and $^{233}U$ are discussed.
The fast reactor concept is currently being intensively studied at Los Alamos with regard to criticality, materials, heat transfer, control, and other phases of reactor design.
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I. INTRODUCTION

This report describes work done at Los Alamos from June to December, 1959, concerning lightweight nuclear propulsion reactors. The conclusions and opinions recorded here are the results of the efforts of numerous people working in the fields of reactor physics, materials, and reactor design.

With fast reactors utilizing Oy or U\(^{233}\) in the form of U\(_{0.2}\)-W cermets or UC-ZrC solid solutions, it appears technically feasible to develop power densities of the order of 1 Mw per pound in the range of 200 to 2000 Mw. This could open a new area of application for upper stage nuclear rockets.

II. FAST REACTORS

A. General Considerations

As is shown in Appendix A, as one decreases the amount of moderation in a critical assembly, the total mass generally decreases, and systems with little or no moderation may have low weights. Bare critical spheres of U\(^{235}\) and U\(^{233}\) weigh 52 and 16 kg, respectively, and reflection with Be reduces these weights even further. Thus criticality is not a necessarily limiting factor in minimizing reactor weights, and the real problems are seen to be (1) finding materials capable of withstanding the high temperatures and H\(_2\) environment of propulsion reactors and (2) designing a reactor that has structural integrity and yet has heat transfer capable of removing the large quantities of energy required for appreciable thrust. Fortunately, the neutron absorption cross
sections for most elements are uniformly low for the energy spectra of fast reactors, allowing a free choice of materials. For example, W, Ta, and Hf, which have large resonance and thermal absorption cross sections, can be used in bulk when desired for their refractory properties. The similarity of high energy neutron cross sections for most elements adds confidence to the numerical results reported here, and insures that the weights need not be increased by large amounts because of engineering details required for a working reactor. Furthermore, since neutron mean free paths are of the order of several centimeters for all materials at high energies, details of core construction such as the exact disposition of structural material and fuel elements will not affect criticality greatly, making it possible to predict reactor weights knowing only gross quantities of materials and void percentage. The criticality calculations presented (in detail in Appendix A) were done using multigroup transport (SNG) techniques which are based on a wide variety of appropriate experimental critical assemblies. Some care has been taken to approximate the effects of changing from one-dimensional spherical assemblies to right circular cylinders (of appropriate height to diameter ratios) with sides and one end reflected. Heat transfer calculations were made by engineers familiar with nuclear propulsion reactors, and their analyses are presented in some detail in Appendix B. Under the turbulent flow conditions involved, the heat transferred to the gas is in the range of 1000 to 2000 Mw per square foot of cross-sectional flow area. Specific designs were considered with due regard for structural and metallurgical limitations based on the current best
information on the physical properties of the materials involved. We therefore feel that the reactor weights and powers presented are realistic, though definitely not final, and that the technology is feasible to the extent discussed for each reactor type. These results appear to offer a sufficient advance over present capability that there is considerable leeway (factors of 2 or more) in both weight and power before the concept becomes unattractive. A number of special problems associated with fast reactors are considered.

B. \( \text{O}_2 \text{W} \) Reactors*

As we have seen, the primary problem in propulsion reactor development is finding suitable materials. For the Dumbo program, cermet of \( \text{U}_2 \text{O}_3 \) dispersed in Mo and W were extensively studied. The thinness of the foils (~0.005 inches) limited loading to about 20 vol. % \( \text{U}_2 \text{O}_3 \) in the cermet. Early calculations (1,2) indicated Be-reflected fast reactors would be interesting at about 40% loading, and it is now felt that this is feasible in thicknesses of the order of 0.04 to 0.08 inches. The physical, mechanical, and chemical properties of W metal are fairly well known, and methods of fabrication are being investigated. A tentative reactor design based on loaded plates which also serve as structural members with a small additional amount of W metal structure has been selected. Twelve cm of Be at 75% of full density is used as reflector in all fast reactor studies presented here. This provides

* Nomenclature: We use "25" to denote \( \text{U}^{235} \), "0" to denote a mixture of 93.5% \( \text{U}^{235} \) + 6.5% \( \text{U}^{238} \), "23" to represent \( \text{U}^{233} \), and U for uranium in a general sense.
fairly good reflection of the high energy neutrons, with some power peaking at the core edge due to return of thermals. While this peaking may be removed by differential loading of the outer fuel plates or placing a thermal neutron poison at the interface, loading of the reflector is now being considered for this and another reason: the heat transfer from a hot wall to a fluid becomes poor if the fluid is at a very much lower temperature, and so one would like to get an appreciable amount of heat into the gas before it reaches the hot core, raising the gas a few hundred degrees above zero Rankine.

The melting point of W is 3370°C, but its mechanical properties deteriorate at somewhat lower temperatures. Creep under the stress of the pressure drop through the reactor appears at present to be a limiting factor for the current design. For a given engine, both the heat transfer coefficient and flow rate are roughly proportional to the operating pressure, and so one can reduce power to obtain lower pressure drop stresses and thus raise the maximum operating temperature as limited by creep in the metal. The maximum exit gas temperature is thus dependent upon design details and probably lies in the area of 2500 ± 300°C.

Bleedout of uranium will probably not be a problem at these temperatures.

The results of calculations (3) for 40% UO₂-loaded W are shown in Figure 1. There is a possibility of increasing loadings to 50%, and the effects of such an increase are shown also. Table 1 gives approximate weights and powers for several cases. The reactor weight includes not only core and reflector but pressure shell (200-300 pounds) and 100 pounds for miscellaneous items (e.g., control system motors).
### Table 1

**OyO₂-W Reactors**

<table>
<thead>
<tr>
<th>Vol. % OyO₂</th>
<th>Oy (kg)</th>
<th>Core Diam. (in.)</th>
<th>Core + Refl. (lbs)</th>
<th>Misc. + Press. Shell (lbs)</th>
<th>Reactor Total (lbs)</th>
<th>Power (Mw)</th>
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</thead>
<tbody>
<tr>
<td>40</td>
<td>~220</td>
<td>20</td>
<td>2500</td>
<td>400</td>
<td>2900*</td>
<td>1000</td>
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<tr>
<td>40</td>
<td>165</td>
<td>15</td>
<td>2165</td>
<td>300</td>
<td>2065</td>
<td>200</td>
</tr>
<tr>
<td>50</td>
<td>190</td>
<td>16</td>
<td>1700</td>
<td>350</td>
<td>2050*</td>
<td>1000</td>
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<td>50</td>
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<td>14</td>
<td>1250</td>
<td>300</td>
<td>1550*</td>
<td>200</td>
</tr>
<tr>
<td>50</td>
<td>275</td>
<td>20</td>
<td>2450</td>
<td>400</td>
<td>2850</td>
<td>2450</td>
</tr>
</tbody>
</table>

*By interpolation.*
Fig. 1  Weight vs. power for Oy-W reactors. Reactor weight = core + reflector + pressure shell + 100 lb miscellaneous.
C. Carbide Reactors

The carbide reactors present a more complicated picture which is difficult to assess at this time. They appear to offer lower weights and higher performance than cermets, but their technology is virtually unexplored. This class of substances includes the highest melting point solids, which fortunately also appear capable of holding uranium at these high temperatures, thus representing the ultimately highest temperatures which might be obtained with solid nuclear heat exchanger reactors. A certain amount of experience has been gained with one member of this class, ZrC (M.P. = 3525°C), and so our results are based on this material.

As a result of work done for the plasma thermocouple experiment, studies of the UC-ZrC system were conducted. (4) As do most metal carbides, these form a continuous series of solid solutions and have melting points as shown in Figure 2. Although a number of pieces of 50-50 and 30-70 vol. % UC-ZrC have been made (and even subjected to neutron fluxes), their physical and mechanical properties are imperfectly known, especially as they might be affected by the techniques of large-scale production of the material.

The first complication which arises is the question of what compositions should be considered for reactor use. While the melting point and related mechanical quantities increase with increasing ZrC content, so does the reactor weight, and one must choose from a spectrum of possibilities. We have considered 50-50 vol. % (M.P. = 2775°C) as a composition which will give both low weight and good exit gas temperatures.
Fig. 2 Melting points of UC-ZrC solid solutions.
and base most of our results on this. A systematic study has been made for various compositions. (5,6) Because of the lack of information on the mechanical properties of this solid solution, a very tentative conservative design was adopted based on \(0.04'' \times 2'' \times 2''\) flat fuel element plates supported by a W structure which occupies 10% by volume of the core. (7) This introduces the complexity of compatibility of the W structure with the fuel. A W-ZrC eutectic temperature of \(-2800^\circ C\) seems to be the limiting factor as well as creep in the W. Further increase in temperature may require advancing the carbide technology to the point where it can be used structurally as well. The existing information indicates U retention is quite adequate for propulsion reactor use. A materials development program is currently under way at Los Alamos to investigate carbide properties. Reactor weights are appreciably better than the cermet system, being \(-1300\) pounds for \(1000\) Mw and \(900\) pounds for \(200\) Mw, including pressure shell and 100 pounds extra. Figure 3 and Table 2 give a number of results.

Table 2

UC-ZrC Reactors

<table>
<thead>
<tr>
<th>Comp. (Vol.%)</th>
<th>Cy (kg)</th>
<th>Core Diam. (in.)</th>
<th>Core (lbs)</th>
<th>Core + Misc. + Reactor Power (lbs)</th>
<th>Total (lbs)</th>
<th>Power (Mw)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50-50</td>
<td>95</td>
<td>11</td>
<td>400</td>
<td>640</td>
<td>260</td>
<td>900</td>
</tr>
<tr>
<td>50-50</td>
<td>145</td>
<td>15</td>
<td>680</td>
<td>1060</td>
<td>300</td>
<td>1360</td>
</tr>
<tr>
<td>50-50</td>
<td>260</td>
<td>20</td>
<td>1520</td>
<td>2210</td>
<td>400</td>
<td>2610</td>
</tr>
</tbody>
</table>

Let us consider the ultimately best possible performance (in terms
Fig. 3 Reactor weight vs. power. 50-50 vol. % OyC-ZrC fuel; 10 vol. % of core as W structure. Reactor weight = core + reflector + pressure shell + 100 lb miscellaneous.
of exit gas temperature) obtainable from carbide systems, assuming they can be made structurally self-supporting. Both HfC and TaC melt at 3800°C, and a solid solution of composition 4-1 TaC-HfC melts at 3930°C, the highest known melting point. Presumably, relatively small percentages (~20%) of UC (M.P. = ~2450°C) will not drastically alter the physical properties of these compounds, and hopefully the uranium retention will also be adequate. A general examination of high-melting carbides containing UC should be conducted. A greater increase in weight would probably result from decreasing the UC concentration rather than substituting HfC for ZrC. A method of avoiding this weight increase by use of U^{233} is discussed in Section III.C. Criticality calculations are being made for HfC and TaC reactors. Critical assemblies and replacement measurements are under way for UC\textsubscript{n} bare and Be-reflected cylinders. Present reactor design emphasis is on obtaining a workable, moderate temperature (2200°C exit gas) fast reactor to test the concept as soon as possible rather than to try for the highest possible temperature immediately.

D. Problems of Fast Reactors

As with any untried scheme, a number of the problem areas can be defined at the outset. Many are general in nature and applicability, such as the control of a fast reactor, while others depend upon specific designs and materials. We will mention those of the former kind which apply particularly to this class of reactors, rather than review all the problems of nuclear rockets here, and try to point out those of the
latter which appear most significant, indicating avenues of solution. The design of power plants operating in the range of hundreds of megawatts per cubic foot is obviously no mean task and will require much excellent engineering work.

Conceptually, fast reactors differ from Kiwi systems in that the average neutron energy is high, the prompt neutron lifetime is smaller, and they require considerably less non-fissile material. Minimum weight graphite systems have intermediate neutron spectra, about the same amount of fissile material, and relatively few thermal neutrons. Thus, the differences are quantitative rather than qualitative, though one should by no means minimize them. This difference comes in controlling the reactor, especially with regard to prompt criticality.

In all normal operation of reactors, the time dependence is determined not by the prompt neutrons (with lifetimes $10^{-5}$ sec or less) but by the delayed neutrons which have an average lifetime of $14$ sec and an abundance of $0.0075$, giving all neutrons an effective average lifetime of $0.1$ sec. It is the latter number which determines reactor periods for reactors operating below prompt critical, and this number is the same for both fast and thermal reactors. The difference comes at prompt critical. The prompt-neutron lifetime for Kiwi B is estimated to be $5 	imes 10^{-5}$ sec, within a factor of 2, which hardly makes the reactor controllable at prompt critical. The fast reactor prompt-neutron lifetime is much smaller, perhaps of the order of $10^{-7}$ sec. While prompt critical reactor excursions are difficult to predict, the shorter lifetime and lack of a large mass of material to absorb the energy release
make the fast reactor excursion more likely to result in fusion and/or vaporization of a portion of the reactor. Thus, control is an essential problem area for fast reactors.

The hydrogen reactivity and startup problems are similar to those of Kiwi B. The introduction of a mass of liquid hydrogen into the core can cause a large increase in reactivity. While the worth of hydrogen per unit mass is higher for fast reactors, the total void is less because of the small size of fast reactors, and the maximum reactivity rise is about the same for both systems. The worth of the hydrogen apparently varies with position in the core and with reactor type, the tungsten reactors being less sensitive than carbide systems. This problem will have to be solved for any nuclear rocket, though it may be more severe for fast reactors.

We will now mention a number of technical design problems. First, there are few elements which are good neutron poisons at high energies, and the reactor core is at a very high temperature, making control by means of absorbing rods run into the core appear impractical. Several schemes have been considered, including removal of fuel rods, poisoning the side reflector, and moving the inlet end reflector. The last seems the best, being mechanically possible and apparently providing adequate control. There is virtually no temperature coefficient of reactivity other than that due to thermal expansion of the reflector.

It is desirable to have the radial power distribution flat or adjust the gas flow such that the exit gas temperature is uniformly high. A truncated cosine power distribution holds over most of the core, with
a rise at the edge due to slowed-down neutrons returning from the reflector. Variable loading, a thermal neutron shield, or fuel element spacing can cope with the radial distribution. A more serious problem of this type appears at the inlet end, where peaking occurs and the heat transfer mechanism is insufficient to transport all the energy to the cold gas. Loading the reflector with fissile material is being considered to solve both problems simultaneously. The decrease in power density at the unreflected end is what is desired to obtain the maximum exit gas temperature, and this presents no problem.

One must maintain the structural integrity of the core against the accelerations and vibrations attendant to flight and yet have sufficient void and smallness of hydraulic diameters for good heat transfer. The Be reflector must be thermally insulated from the hot core. The carbides appear somewhat brittle at low temperatures, while their high temperature properties are still largely unknown. Tungsten is limited by creep at high temperatures. These and other engineering difficulties must be overcome before fast reactors are a reality.

III. U$^{233}$ REACTORS

A. Introduction

The neutronic properties of U$^{233}$ are much superior to those of Oy, as can be seen from comparison of the bare sphere critical masses of 16 and 52 kg, respectively. Thus, fast reactors can be scaled down in weight by factors of 2 to 3 when the Oy is replaced by 23, as has been
confirmed by calculations using the best available multigroup cross sections for $^{233}U$. This superiority extends to the intermediate energies of minimum weight graphite reactors, which can be halved in weight with $^{23}$, as has been pointed out. (8,9) Lighter weight, high temperature carbide reactors may be made best with $^{233}U$.

The major disadvantage of $^{233}U$ is the activity of a contaminant ($^{232}U$) associated with its manufacture, which results in greater difficulty and expense in its production and use.

There are ways to reduce this contaminant or to handle the material while its activity is low, as will be discussed in Section III.E. However, since its chemical, physical, and mechanical properties are identical to those of $^{25}$ and $^{28}$, these materials can be used to develop most of the technology required. The $^{233}U$ reactors will follow as a direct extension of the Oy reactor experience. It is important to recognize now the capabilities that nuclear reactors can have later.

The fast reactor and $^{233}U$ developments should extend the possible domain of nuclear heat exchangers into areas where they have not been given much consideration because of the assumed large minimum reactor weight.

B. $^{23}O_2-W$ Reactors

Using the same design principles as in Oy-W reactors, $^{233}U^{23}O_2-W$ reactors can be made at one-half to one-third the weight at the same power. Otherwise, statements concerning the cermet reactors hold equally well here. The weight reductions should be independent of the
particular design. Results for two cases of 40% loading are given in Table 3 and Figure 4, together with estimates for the 50% loading. The 40% loading gives a weight-power relation close to that for OyC reactors. Typical values are 1400 pounds total for 1000 Mw and 830 pounds total for 200 Mw.

Table 3

<table>
<thead>
<tr>
<th>Vol.% 23 O₂ (kg)</th>
<th>Core Diam. Approx. (in.)</th>
<th>Core + Refl. Press + Shell Total (lbs)</th>
<th>Power (Mw)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>86</td>
<td>1070 330 1400</td>
<td>1000</td>
</tr>
<tr>
<td>40</td>
<td>44</td>
<td>610 220 830</td>
<td>200</td>
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<tr>
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<td>~75*</td>
<td>620* 960*</td>
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</tr>
<tr>
<td>50</td>
<td>~37*</td>
<td></td>
<td>1000</td>
</tr>
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</table>

*Estimate based on Oy 40% vs. 50% loadings.

C. U²³³ Carbide Reactors

One may use U²³³ in carbide reactors either to further reduce their weight or to obtain higher melting point fuel elements.

Consider weight reduction first. The results indicate it is possible to make reactors (50-50 230-ZrC, 10% W) weighing only 950 pounds for the 1000 Mw (50,000 pounds thrust) level and to make 200 Mw reactors weighing only 500 pounds (including pressure shell and 100 pounds for miscellaneous extras). These weights are so low as to...
Fig. 4 Reactor weight vs. power for $^{233}\text{U}_2\text{O}_2$ reactors. Reactor weight = core + reflector + pressure shell + 100 lb miscellaneous.
effectively eliminate reactor weight as a serious limitation on the use of nuclear rockets. Even the weight of shields, if such are required, is reduced considerably, owing to the small reactor size.

The OyC reactors are already so light in weight at 1000 Mw that, rather than save only a few hundred pounds by using 23, one can use it to raise the melting point several hundred degrees and thus greatly improve specific impulse, assuming the structural problems of all-carbide cores or the W compatibility problems have been solved. A calculation for the 1000 Mw sized reactor has shown that the 50-50 OyC-ZrC fuel (M.P. = 2775°C) is neutronically equivalent to an equal volume of 22 vol. % U^{233}C-78% ZrC (M.P. = ~3090°C), which melts 315°C higher. This mixture weighs only 81% of the equivalent Oy fuel. Similar results may be expected with other refractory carbide solid solutions. By comparing the weight-power relationships, we can see that these low weight carbide reactors, including those using Oy, are heat-transfer-limited and that reducing their weight by any appreciable fraction decreases their power correspondingly. The results are shown in Table 4 and Figure 5.

Table 4

\[ \text{U}^{233}\text{C-ZrC Reactors} \]

<table>
<thead>
<tr>
<th>Vol.% 23C (kg)</th>
<th>Core Diam. (in.)</th>
<th>Core (lbs)</th>
<th>Core + Refl. (lbs)</th>
<th>Misc. + Press. Shell (lbs)</th>
<th>Reactor Total (lbs)</th>
<th>Power (Mw)</th>
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<td>50 42 10</td>
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<td>320</td>
<td>180</td>
<td>500</td>
<td>200</td>
<td></td>
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<tr>
<td>50 86 15</td>
<td>420</td>
<td>690</td>
<td>260</td>
<td>950</td>
<td>1000</td>
<td></td>
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<tr>
<td>22* 42</td>
<td>500</td>
<td>250</td>
<td>750</td>
<td>200</td>
<td></td>
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<tr>
<td>22* 64</td>
<td>870</td>
<td>300</td>
<td>1170</td>
<td>1000</td>
<td></td>
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* Results estimated from 50-50 OyC reactors. The effect of increased exit gas temperature upon power level is neglected. 10% W structure is retained.
Fig. 5 Reactor weight vs. power. $^{233}\text{U-ZrC}$ fuel; 10 vol. % of core as W structure. Reactor weight = core + reflector + pressure shell + 100 lb miscellaneous.

19
D. \( U^{233} \) in Graphite Reactors

Present designs for Kiwi B are based on having half the graphite loaded with \( \sim 400 \text{ mg/cc of } OyO_2 \), while half is unloaded and used structurally. This represents C/U\(^{235} \) ratios of 80 for the loaded portions and about 160 overall. As has been pointed out, (10) if one could load all the graphite, the reactor weight could be reduced to 75% of the present value, or to about 6000 pounds. It has been observed (8,9) that the weight of a given design can be halved by the use of \( U^{233} \) in place of Oy at the same loading. Using the ratios derived from Figure 6 for solid graphite spheres (these ratios are expected to hold approximately for cases with void and reflection), we can estimate minimum weights for various graphite reactors. The results (Table 5) for power are nominal values scaled to a power range expected for the Kiwi B weight.

Table 5
Graphite Reactors

<table>
<thead>
<tr>
<th>Fuel</th>
<th>% Loaded Graphite</th>
<th>Average C/U</th>
<th>Approx. Weight (lbs)</th>
<th>Power Range (Mw)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>50</td>
<td>160</td>
<td>8000</td>
<td>1000-2000</td>
</tr>
<tr>
<td>25</td>
<td>100</td>
<td>80</td>
<td>6000</td>
<td>800-1600</td>
</tr>
<tr>
<td>23</td>
<td>50</td>
<td>160</td>
<td>4000</td>
<td>650-1300</td>
</tr>
<tr>
<td>23</td>
<td>100</td>
<td>80</td>
<td>3000</td>
<td>500-1000</td>
</tr>
</tbody>
</table>

E. Problems Associated with \( U^{233} \)

\( U^{233} \) differs from oralloy in that it must be made artificially at
\[
\frac{M_T(C/U=80)}{M_T(C/U=160)} = \begin{cases} 
.74 \text{ FOR } 25 \\
.77 \text{ FOR } 23
\end{cases}
\]

\[
\frac{M_T(23)}{M_T(25)} = \begin{cases} 
.46 \text{ FOR } C/U=160 \\
.48 \text{ FOR } C/U=80
\end{cases}
\]

Fig. 6 Total and fissile masses for uranium-loaded solid graphite spheres using U\textsuperscript{235} and U\textsuperscript{233}.
the cost of one neutron per atom and that it is radioactive both naturally and due to associated impurities. Details may be found in reports on $^{233}$U, (11,12) but the main features will be reviewed here.

Pure $^{233}$U has a soft $\gamma$ component of about 15 kev giving a surface dose (excluding the $M$ x-ray component) of about 1.5 rep/hr, which can be reduced to 0.1 rep/hr with gloves and a few mils coating. The 1.5 rep/hr value is comparable to 2.2 rep/hr for Pu under similar circumstances.

The real difficulty is a 2.6 Mev $\gamma$-ray from the decay products of the $^{232}$U impurity formed with the $^{233}$U. Briefly, this decay chain ($4n$ series) is

$$
^{232}U \xrightarrow{\alpha} ^{228}Th \xrightarrow{\alpha} ^{224}Ra \xrightarrow{\alpha, \beta_{\text{fast}}} ^{212}Bi \xrightarrow{\beta_{\text{fast}}} ^{208}Tl \xrightarrow{\beta_{\text{fast}}} ^{208}Pb
$$

The $^{208}Pb$ is formed from $^{208}Tl$ only in its 2.6 Mev excited state and is the source of the trouble. The 1.9 yr half-life of $^{228}Th$ controls the buildup of the activity for short times ($\leqslant 5$ yr). The activity thus builds up (linearly for about one year) from zero at the time of separation (casting date) of the U from the $^{232}Th$ and $^{228}Th$. The amount of $^{232}U$ impurity is strongly dependent upon the conditions of formation of the $^{233}U$ and the methods of separation and ranges from 2 to $10^3$ parts per million. The major mode of formation of $^{232}U$ is from the reaction $^{232}Th(n,2n)Th^{231}$ followed by $\beta$ decay and neutron capture. The $n,2n$ reaction has a threshold of $6.35$ Mev, and so could be considerably
reduced by separating the breeding material from the fuel by a moderator. Existing small quantities of $^{233}U$ containing 2 to 10 parts per million $^{232}U$ were probably made by separating $^{233}Pa$ (27 day half-life) and letting it decay to $^{233}U$. $^{232}Pa$, the parent of $^{232}U$, has a 1.3 day half-life.

Thus, to minimize the activity of $^{233}U$ and its $^{232}U$ contaminant, one can control its method of formation and can process it into fuel elements quickly to avoid the buildup associated with $^{238}Th$ formation. Once the reactor has been run at power, the fission product activity will dominate.

However, while individual fuel elements will be capable of being handled prior to a power run, the reactor as a whole will be somewhat active, and it may be difficult to work in contact with the core. Normally, the reflector and self shielding should eliminate the necessity for heavy shields around the reactor. In the event of a launch pad accident for a cold reactor, $^{233}U$ would be a much greater problem than $^{235}U$ because of the attendant activity. The extent of this problem depends upon the likelihood that the core material would be dispersed (e.g., by the chemical explosion or possibly even combustion of the fuel elements themselves).

$^{233}U$ and Pu have smaller fractions of delayed neutrons than $^{235}U$, the values being .30, .23, and .75 per 100 fission neutrons, respectively. The distribution of precursors results in average lifetimes for delayed neutrons of 16.7, 14.4, and 14.1 sec, respectively, for $^{233}U$, Pu, and $^{235}U$ which, with the abundance of delayed neutrons, gives average
lifetimes of all neutrons of 0.05, 0.033, and 0.106 sec.

While this makes control more difficult for $^{233}\text{U}$ or Pu fueled reactors, it does not make it infeasible, as evidenced by the existing Pu fast critical reactors and assemblies.

Finally, $^{233}\text{U}$ is not available in quantity, and several years and considerable expense would be required for its production. Because of these considerations, it appears that the state of the nuclear propulsion art will be advanced to a working level before $^{233}\text{U}$ is actually used. A second-stage engine might be the most likely candidate for its initial test, especially since upper stages are the most useful domain of small reactors, and the light weight would allow it to be put on relatively small boosters (Thor or Atlas).

IV. DISCUSSION

A. The Function of Lightweight Reactors

The domain of lightweight reactors can be seen in perspective by comparison (Figure 7) with the graphite system which leads to Condor, a 10,000 Mw reactor weighing about 20,000 pounds. Condor is capable of placing 25,000 pounds of payload into orbit with a 400,000 pound single stage, a mission equivalent to that of the three-stage Saturn-Titan-Centaur rocket. Kiwi B is, to a certain extent, a small-scale model having one-fourth the power-to-weight ratio of Condor. Thus, Kiwi B bears the same relation to Condor as the 200 Mw fast reactors do to the 2000 Mw class. The smaller models, while not optimum for their
Fig. 7 A comparison of the weight-power domains of fast reactors and graphite systems.
type, nevertheless can perform useful functions.

We see that for graphite reactors, the weight cannot be appreciably reduced below that for the 1000 Mw reactor, and this value is then a rough lower limit for the useful power range, other things being equal. This same state is reached for the fast reactors at about 200 Mw. We have done very few mission studies thus far and cannot comment at length upon the possibilities of such a power level, but a general study (13) of missions starting from earth orbit has indicated that thrust-to-mass ratios of 0.1 are quite adequate under many circumstances. Thus, a 200 Mw reactor (10,000 pounds thrust) could be used to propel a 100,000 pound stage which has been placed in a low earth orbit. The thrust level is high enough for soft landings of smaller vehicles or after a large portion of the propellant has been used as opposed to that of ion or plasma rockets, which have very low thrusts. Fast reactors appear at present to be quicker to develop than electrical systems. One thing can be gained by reducing the power, which is higher exit gas temperature and consequently higher specific impulse. The 1000-2000 Mw level appears best for fast reactors, their having a thrust of 50 times their weight. It is difficult to say what the upper power limit is for such reactors. One may be limited by structural considerations to a maximum void fraction of about 0.6, which limits the size for a given core composition.

We have investigated some applications (14, 15, 16) for fast reactors in the 1000-4000 Mw level and find that an increase in payload of about 5000 pounds over Kiwi powered rockets is possible at the same
exit gas temperature, simply due to the lighter weight, with further increases if better performance can be obtained from fast systems. One application for which Kiwi powered rockets seem marginal, i.e., as second stages on Atlas-class boosters, seems ideal for fast reactors in the power range of 600-1300 MW. Such a combination (Atlas + fast reactor stage) might equal the 1,000,000 pound Saturn three-stage chemical rocket performance, at much lower cost.

For small vehicles, radiation at the payload may be an important factor. This depends strongly upon the vehicle geometry and duration of the run, so it is difficult to make quantitative statements without a particular situation in mind. However, some general statements can be made based on estimates for a 1000 MW, 300 sec run and some general considerations. Radiation effects in an unmanned rocket will be negligible except possibly to semiconductors and, of course, radiation detectors. The propellant acts as a shadow shield, and the total dose at the payload is very sensitive to the amount left in the tank at reactor shutdown time. In a manned vehicle the biological dose for this case is of the order of $10^3$ rads, and some form of shielding would be necessary even for a larger rocket. Manned rockets would probably have a storable chemical final stage, and its propellant could be utilized as a biological shield. Because of their higher density and lower weight, fast reactors have much smaller diameters than Kiwi B and would thus require much less shield weight should shielding be necessary.
B. The Development of Lightweight Reactors

A comparison of the various types of lightweight reactors (Figure 8) gives only part of the picture. We must also consider the time and expense necessary to develop each type and its ultimate possibilities in terms of performance.

The Oy-W systems appear at present (December, 1959) to be easiest and quickest to develop because of the experience in fabrication and the knowledge of the mechanical properties of the material. They have an absolute upper limit of $3370^\circ C$ (M.P. of W) for the exit gas temperature, and a practical limit $300-1000^\circ C$ less than that owing to structural limitations.

The UC reactors, though lighter in weight, suffer from lack of information concerning the physical properties of carbide solid solutions, even of the UC-ZrC system. As presently conceived, they require W structural support members, which limits their achievable temperature. Development of carbide technology to the point where the carbide is self-supporting and the use of HfC-TaC-UC solid solutions promise the ultimate performance for solid nuclear heat exchanger rocket engines, having an absolute upper limit of $3930^\circ C$ (M.P. of 1 HfC-4 TaC).

The value of high exit gas temperature can be seen as follows. Almost any practical nuclear rocket engine will operate at $2000^\circ C$, giving a specific impulse of about 750 sec compared to 375 for very good chemical rockets. For a simple gas, the specific impulse is approximately proportional to the square root of the temperature, and so increasing $T$ to $3000^\circ C$ will raise the impulse to near 900 sec. At this
Fig. 8 A comparison of weights for the various reactor types. Weights include pressure shell + 100 lb miscellaneous.
point, hydrogen dissociation begins to enter the picture, and by
3500°C, with appropriate pressures, the specific impulse might be
raised to 1050 sec or more. This is the motivation for seeking the
ultimate in exit gas temperature.

On a short-term scale, we seek to demonstrate the feasibility of
fast reactors as quickly as possible. In 1960, we hope to evaluate
carbides and cermets and decide which offers the best opportunity for
an early feasibility demonstration. While studies will continue on
both types of materials, the main light-reactor program will aim towards
construction of a 1000 Mw fast reactor which will be first tested at
the 100 Mw level. This plan represents a considerable extrapolation
and is quite subject to change.

It is relatively easier to evaluate the situation with regard to
the use of U$^{233}$ in propulsion reactors. It is clear that, where a con-
cept employing Oy works, the corresponding reactor using U$^{233}$ will also
be feasible and offer considerable weight and/or performance advances.
It is equally clear that complications arise owing to the activity and
availability of the material. The activity of U$^{233}$ and its contaminant
U$^{232}$ necessitates additional cost and effort in fabrication, makes sub-
sequent manipulations in the core more difficult, and adds the problem
of contamination from accidents prior to full power operation.

Thus, it would be much easier to use U$^{233}$ in a proven reactor type
than to develop a new concept with it as an added complication. Never-
theless, these problems are by no means insurmountable. The Kiwi
system is an important case in point. Much of the technology has
already been developed and to some extent, the Kiwi A test proved feasibility. A continuing series of Oy-fueled test reactors leading to a breadboard engine will further advance the technology, and it is hoped will lead to an early flight demonstration of a graphite reactor. An 8000 pound 1000-2000 Mw reactor is only marginally useful as a single-stage, ground-launched rocket or as a second stage on an Atlas or Titan. At 4000 pounds, which is what a U^{233}-fueled reactor would weigh, one might expect a considerable improvement in the capability of a nuclear rocket, especially as a second stage on Titan or a larger booster. This capability should be explored and evaluated in the light of the added problems associated with the use of U^{233} and the possibility of the smaller Oy-fueled fast reactors becoming operational in time to compete with graphite systems. On one hand, we have at least a two-year lead time required before large quantities (~100 kg) of U^{233} could be produced, while on the other, the fact that fast reactors are as yet completely untried. One should keep the possibility of a U^{233}-graphite reactor under continuous scrutiny, for while it does not appear optimum in the long run, it may represent a useful rocket reactor over an intermediate period, especially if lighter systems are late in materializing. It may be worth pointing out here that Pu, while neutronically slightly superior to U^{233} and much more readily available, does not have as favorable physical and chemical properties as uranium (e.g., melting points of oxides and carbides) and would require development of a new technology (e.g., loading of Pu oxide in graphite) with active material.
The most likely Pu reactor might be one with PuO$_2$ (M•P• 2240°C) dispersed in W, similar to UO$_2$-W cerments. We have examined the criticality of such PuO$_2$ cerments with 35, 25, and 20 vol. % loadings (17) and find them good possibilities, even up to 10,000 Mw or more if the PuO$_2$ can be retained in the fuel element in a molten condition. The cost of the Pu for the latter case (10,000 Mw, 500 kg Pu) is small (~$8,000,000) compared to the fabrication cost of a large chemical booster which could do an equivalent job, and also small compared to other costs for a nuclear missile. Thus actual cost for fissionable material is not prohibitive.

In the long view, U$^{233}$ appears as a natural choice as the fuel for lightweight reactors as a natural consequence of experience with Oy fast reactors. Hopefully, the thorium breeding program will eventually result in extensive experience with U$^{233}$ and production of the large quantities required for economic consumption of the world’s supply of fertile material.

V. SUMMARY

1. The fast reactor concept leads to lightweight nuclear rocket reactors for power levels of 200-2000 Mw. Power densities of the order of 1 Mw per pound open a new area for upper stage use of nuclear rockets.

2. Because of the uniformity of high energy cross sections, fast reactors may employ almost any element in bulk, allowing the use of the most refractory materials and leading to high exit gas temperature and specific impulse.
3. Two types of fuel materials are being examined for such use: UO$_2$-W cerments and UC-ZrC solid solutions. The cermet fuel has received more study to date and seems capable of quicker development. The carbide materials offer much lower weights and higher temperatures but suffer from a lack of information on their physical and mechanical properties. Carbides include the highest melting solids known, offering the ultimate performance for solid nuclear heat exchanger rockets.

4. The main problem area for fast reactors is in control. While fast reactor neutron kinetics are the same as those for thermal reactors while operating below prompt critical, excursions above prompt critical are more likely to lead to fusion or vaporization due to the shorter period and the smaller amount of material present to absorb the energy released.

5. The use of U$^{233}$ in place of O$_{235}$ can reduce reactor weights by factors of 2 to 3 for intermediate and fast reactors or, alternatively, can be used to increase the exit gas temperatures for lightweight carbide reactors.

6. Because of radioactivity associated with U$^{233}$ and its contaminant U$^{232}$, it is somewhat more difficult and more costly to use, especially in development work. Thus, U$^{233}$ is likely to be used in fast reactors only after they have been made operational with U$^{235}$ as the fissile material.

7. The use of U$^{233}$ as a fuel in small (1000 Mw) graphite reactors can reduce their weight to 3000-4000 pounds, making them much more useful as motors for second stage vehicles. One must consider the lead
time required for production of the necessary $^{233}$U if such reactors
are to be constructed in a useful time period.

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APPENDIX A

CRITICALITY AND WEIGHT

The Role of Moderation

Figure A1 is a plot of mass (both total and fissile material) vs. diameter for bare solid spheres of graphite uniformly loaded with U^{235} (Al) with the moderation (C/U ratio) as a parameter. This behavior is representative of all light element moderators (with the exception of hydrogen). Starting with a sphere of solid uranium, the critical mass (fissile material) increases with the addition of C, as the moderation lowers the neutron energies from the fast region (neutronically favorable) toward the intermediate energy resonance region, where the average capture to fission ratio is higher. Other effects such as moderator scattering and dilution of the fissile material (which affect the leakage probability) also play a role in this region. The high energy capture in the light moderator is usually very small, but heavy elements will be significant in this respect. The critical mass goes through a maximum as the scattering reduces leakage and the moderation lowers the average neutron energy toward regions of lower capture to fission ratios. A minimum is reached where the neutrons are almost completely thermalized and the critical mass rises again, owing to absorption by the very large amount of moderator present and in the fissile material itself. The important point is that the total mass increases monotonically with moderation.
Fig. A1  Bare solid spheres of U$^{235}$ in graphite; C/U as parameter.
There are several items in a reactor which affect the total mass relationship quantitatively, but not qualitatively, such as void, reflection, and non-uniform loading. Addition of void for gas passages increases the total and critical masses inversely as the square of the average density for bare reactors. Reflection decreases the critical mass and is most effective for small reactors, where the total mass may be appreciably decreased, especially with a low density reflector (such as Be) on a dense core. Non-uniform loading can affect masses in either direction, though in propulsion reactors where one tries to flatten the power distribution this results in increased critical masses.

We see, then, from Figure A1 that to have a lightweight reactor we would like as high a ratio of fissile to moderating material as possible within the constraints of material, structural, and heat transfer considerations. The optimum regime of graphite propulsion reactors is in relatively large systems (of the order of 10,000 Mw and 20,000 pounds), but many conditions caused the Laboratory to begin with a graphite reactor as small as practicable. It was found possible to make fuel elements with loadings representing C/U ratios of about 80, and the need for unloaded graphite structure in the present design (Kiwi B) increased the average C/U ratio to about 160. Note that this causes Kiwi to be an intermediate rather than thermal reactor, requiring a relatively large critical mass (~140 kg Ox in practice). This design is limited by the maximum loading of graphite, and to go to the lighter weights theoretically possible requires a new fuel material, e.g.,
a carbide solid solution. Ideally, pure solid fissile material would give the lowest weight, but materials, gas void, and heat transfer area force a compromise at higher C/U ratios. This must be examined individually for each material; e.g., Zr has a low absorption cross section in the resonance and fission energy regions, while Hf has appreciable absorption, and large HfC/UC ratios may not be practical.

**Basis of Calculations**

Most of the calculations were made using a one dimensional $S_4$ transport code employing 20 to 40 space points and 13 energy groups.

The cross sections employed were checked against a wide variety of experimental critical assemblies (A2) and replacement measurements in fast spectra. (A3) These included Oy spheres, bare and reflected with $^{238}U$, Be, W, etc., and mixtures of Oy and C. Uranium and Zr cross sections were transcribed from Hansen's work, (A4) and various theoretical and empirical correlations were used where data was insufficient.

The cloudy crystal ball model (A5) was used to obtain high energy transport cross sections and the statistical model for inelastically scattered neutron distributions. (A6) The energy groups covered energies down to thermal, and they are listed in Table A1, along with the spectra and fission rates for a typical fast reactor. By comparison with an Oy sphere reflected with 2 inches of Be, we see that the propulsion reactors are truly fast.
## TABLE A1

Energy Distribution of Neutrons and Fissions

<table>
<thead>
<tr>
<th>Energy Groups</th>
<th>50-50 Vol. % UC-ZrC, Be-reflected Reactor; 10% W</th>
<th></th>
<th>Core Spectrum (%)</th>
<th>Reflector Spectrum (%)</th>
<th>Fissions/Group (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>10 Mev</td>
<td>2.2</td>
<td>3.3</td>
<td>1.7</td>
<td>5.4</td>
</tr>
<tr>
<td>Group 2</td>
<td>3.7</td>
<td>10.5</td>
<td>15.3</td>
<td>8.2</td>
<td>23.0</td>
</tr>
<tr>
<td>Group 3</td>
<td>Fast</td>
<td>1.3</td>
<td>17.9</td>
<td>16.6</td>
<td>27.6</td>
</tr>
<tr>
<td>Group 4</td>
<td>0.5</td>
<td>18.2</td>
<td>26.2</td>
<td>17.3</td>
<td>18.8</td>
</tr>
<tr>
<td>Group 5</td>
<td>0.183</td>
<td>20.0</td>
<td>19.2</td>
<td>16.6</td>
<td>11.9</td>
</tr>
<tr>
<td>Group 6</td>
<td>24 Kev</td>
<td>12.0</td>
<td>5.8</td>
<td>12.7</td>
<td>5.0</td>
</tr>
<tr>
<td>Group 7</td>
<td>3</td>
<td>6.4</td>
<td>1.5</td>
<td>8.9</td>
<td>2.4</td>
</tr>
<tr>
<td>Group 8</td>
<td>Resonance</td>
<td>0.45</td>
<td>4.5</td>
<td>6.1</td>
<td>1.9</td>
</tr>
<tr>
<td>Group 9</td>
<td>61 Ev</td>
<td>2.6</td>
<td>0.08</td>
<td>4.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Group 10</td>
<td>8</td>
<td>1.2</td>
<td>0.10</td>
<td>2.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Group 11</td>
<td>1</td>
<td>1.5</td>
<td>0.025</td>
<td>1.8</td>
<td>0.7</td>
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<tr>
<td>Group 12</td>
<td>Epithermal</td>
<td>0.1</td>
<td>1.0</td>
<td>0.005</td>
<td>1.2</td>
</tr>
<tr>
<td>Group 13</td>
<td>0.025</td>
<td>1.8</td>
<td>0.006</td>
<td>2.2</td>
<td>1.0</td>
</tr>
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</table>
The one dimensional transport code was used for straightforward calculations such as variation of core composition, reflector thickness and density, fission distributions, reactivity worth of H₂, etc. To check a few of the more critical points, i.e., the control worth of a movable inlet end reflector and the extra core material required in the absence of the exit end reflector (and to check the conversion from spheres to cylinders) we ran a series of two dimensional S₄ transport calculations. (A7) In order to conserve machine time, we collapsed the 13 group cross sections to 4 groups, based on the neutron spectra obtained from the one dimensional calculations. The 4 group constants gave satisfactory results for the reactivity (k = 1.006 compared to 1.000 for 13 groups) for the one dimensional check case. The two dimensional cases (Figure A2) showed that our buckling conversion from sphere to cylinder was conservative, that our estimate for reflector savings was adequate, and that enough reactivity change could be induced by movement of the inlet end reflector for its consideration as a means of control. Power distributions for final designs can also be evaluated with this code.

Some representative calculations for spheres are presented in Table A2 to give the data before corrections were made for non-sphericity, removal of the exit reflector, etc. Experiments are in progress to check the validity of these computations.
A. 1 dim. check on 4 group cross sections

4 group, 1 dim. $k = 1.006$
13 group, 1 dim. $k = 1.000$

B. Completely reflected (9 cm Be) cylinder

$k = 1.038$

C. Exit reflector removed; replaced by 12 cm of core

$k = 1.033$

Core composition at./cc $\times 10^{-24}$

- $^{235}U$ .00588
- $^{238}U$ .00040
- C .0139
- Zr .0076
- Mo .0064

D. Movement of inlet reflector

$k = .971$
$\Delta k = .062$
$= .007$/cm, $\sim$1/cm

FIGURE A2 TWO DIMENSIONAL CASES
### TABLE A2

Representative Calculations

<table>
<thead>
<tr>
<th>Core (Vol. %)</th>
<th>Reflector (% Full Density)</th>
<th>$R_{\text{core}}$(cm)</th>
<th>$t_{\text{refl.}}$(cm)</th>
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<tbody>
<tr>
<td>OyC</td>
<td>ZrC, Mo or W</td>
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<td></td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>10 Mo</td>
<td>98</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>10 Mo</td>
<td>98</td>
</tr>
<tr>
<td>15</td>
<td>25</td>
<td>10 Mo</td>
<td>98</td>
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<td>25</td>
<td>10 Mo</td>
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<tr>
<td>15</td>
<td>25</td>
<td>10 W</td>
<td>98</td>
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<table>
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<tr>
<th>Oy$_2$O$_2$</th>
<th>W</th>
<th>Loading (%)</th>
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<td>15</td>
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<td>35</td>
<td>50</td>
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<table>
<thead>
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<th>U$^{233}$C</th>
<th>ZrC, W</th>
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APPENDIX A REFERENCES


APPENDIX B

METHODS USED IN PERFORMANCE ANALYSIS

The general problem of determining the relationship between reactor size and performance characteristics is not one which can be solved quickly with precision. However, if one is content to deal with averages and approximations with the realization that the detailed behavior and design problems of the reactor system may require considerable modification of the results, very simple and straightforward analyses are possible.

It is assumed that reactor power is limited by:

1. Core exit Mach number
2. Fuel element internal temperature drop
3. Fuel element surface temperature
4. Reactor core pressure drop.

It is, of course, not always possible to assign the correct maximum value to each of these quantities for a variety of reactor configurations, so it is necessary to make rather arbitrary choices based on a best guess as to what constitute reasonable values.

Mach Number

The core exit Mach number is given by

\[ M = \frac{V}{(\gamma g \cdot RT)^{1/2}} = \frac{V}{A_f P_e (RT/\gamma g)^{1/2}}. \]

*Symbols used here are defined at the end of the appendix.*
If exit gas temperature and Mach number are now fixed, then

\[ \dot{w} = C_1 \dot{P}_e \]

and since power is proportional to flow rate for fixed inlet and outlet temperatures, the Mach number limited power of a particular reactor with known flow area is

\[ Q = C_2 P_e. \]

In the analysis of the various fast reactors, the exit Mach number was taken as 0.261, the exit gas temperature as 4000°F, and the weight flow to power ratio as 0.07 lbs/Mw-sec. Various exit pressures were used.

Fuel Element ΔT

It is assumed that the fuel elements are thin flat plates of uniform thickness and spacing. The flat plate equation:

\[ \Delta T = \frac{Q''t^2}{8k_f} \]

taken for \( Q'' = \frac{Q}{v_f} \)

may be rewritten to obtain

\[ Q = \frac{8k_f v_f \Delta T}{t^2} \]
which for a given reactor with fuel conductivity known or assumed
and a given minimum fuel thickness becomes

\[ Q = C \Delta T. \]

Thus, if one knows or is willing to guess an acceptable \( \Delta T \), the internal
temperature drop limited power can be ascertained. Values for these
quantities which were used in the fast reactor studies were: UC-ZrC
thermal conductivity, 16 BTU/hr-ft-°F;* W-\( \mathrm{U}_2 \) thermal conductivity,
20 BTU/hr-ft-°F; minimum plate thickness, .040 inch. Values of \( \Delta T \) up
to 500°F were studied.

Fuel Element Surface Temperature

The accurate determination of the maximum fuel element surface
temperature requires a knowledge of radial and axial power profiles and
the coolant flow distribution, and even then usually requires extensive
calculation. Again, however, approximate results can be quickly obtained
using the following empirical expression (Ref. 6 of text) which ignores
many variables which have a more or less negligible effect.

*This value was based on conductivities of pure UC and pure ZrC. Very
recent measurements indicate conductivity for solid solutions which are
much lower (half the above value), as is the case with disordered metal
alloys for which the resistivity (both electrical and thermal) displays
a parabolic variation \((x - x^2)\) with composition \((x)\). Since thermal
conductivity is frequently the property which limits the power for a
given design, and the power is a linear function of the conductivity
for a given plate thickness, the results presented in the body of this
report are quite sensitive to this parameter. Thus different designs,
thinner plates, variable loading or other techniques might be required
to achieve the heat transfer estimated in the body of this report for
UC-ZrC reactors. (The above footnote was written by Ralph S. Cooper.)
\[
\frac{T_w - T_{g_1}}{T_w - T_{g_2}} = e^{\frac{L}{120D}}
\]

Thus maximum wall temperatures can be computed for each reactor for which core length and hydraulic diameter are known.

Core Pressure Drop

Since pressure drop depends on both flow rate and pressure level, it is not convenient to obtain an independent formula linking power level to pressure drop for each reactor. The alternate approach is to calculate approximate pressure drops for power levels which are otherwise determined, i.e., Mach number or temperature limited power, using reasonable values of pressure. The formula used for these calculations was:

\[
\Delta P = \frac{0.002 \ D \ C^2}{m^p} \ln \frac{\rho_1}{\rho_2} + \frac{0.008 \ L}{D}.
\]

Pressure drops of 200 to 300 psi were considered tolerable with average pressures near 1200 psi.

Example

To illustrate the use of these formulas, we consider a UC-ZrC reactor with core composition 20 v/o UC, 20 v/o ZrC, 10 v/o W, and 50 v/o void. Nuclear calculations establish the core dimensions as 20.1 cm radius and 54.4 cm length or 15.8 inches diameter by 21.4 inches long. The cross-sectional flow area is then 98.3 in\(^2\).
The Mach number limited power at 1000 psi exit pressure is found to be 1400 Mw. The $\Delta T$ limited power is 984 Mw for $\Delta T = 300^\circ R$ and 1640 Mw for $\Delta T = 500^\circ R$. The maximum wall temperature is calculated to be $4780^\circ R$.

If we assume that the fuel material can withstand a $\Delta T$ of $500^\circ R$, we see that the maximum power for this reactor is 1400 Mw, the smaller of the Mach number and $\Delta T$ limited values. At this power level the core pressure drop is calculated to be 305 psi for an average pressure of 1150 psi.

**Summary**

It is quite clear that the many assumptions and approximations which are contained in the methods of analysis leave considerable room for doubt in the results. However, as is pointed out in the body of this report, there is considerable margin for error in the calculations before these concepts cease to be of interest. Furthermore, a number of considerably more rigorous techniques which have been employed to evaluate the reactors of immediate interest, the 1000 Mw class, indicate that the approximate methods described here yield reasonably valid results in this power range.
LIST OF SYMBOLS USED IN APPENDIX B

$A_f$ Cross-sectional flow area, $ft^2$

$C, C_1, C_2$ Constants

$D$ Hydraulic diameter, $ft$

$G$ Mass velocity, $lbs/ft^2$-sec

$g$ Conversion factor, $32.2~ ft/sec^2$

$k_f$ Fuel thermal conductivity, $BTU/hr-ft-^\circ F$

$L$ Core length, $ft$

$M$ Mach number

$m$ Molecular weight

$P_e, \bar{P}$ Core exit pressure, core average pressure, $psi$

$\Delta P$ Core pressure drop, $psi$

$Q$ Reactor power, $Mw$ or $BTU/hr$

$Q''$ Power density, $BTU/hr-ft^3$

$R$ Gas constant

$T, \Delta T$ Temperature, temperature difference, $^\circ R$

$T_m$ Mean gas temperature in core, $^\circ R$

$T_{\text{g}_1}, T_{\text{g}_2}$ Gas entrance and exit temperatures, $^\circ R$

$T_w$ Maximum fuel surface temperature, $^\circ R$

$t$ Fuel plate thickness, $ft$

$V$ Gas velocity, $ft/sec$

$v_f$ Fuel volume, $ft^3$

$\dot{W}$ Mass flow rate, $lbs/sec$

$\rho$ Gas density, $lbs/ft^3$

$\gamma$ Ratio of specific heats