Evaluation of the $d+t$ Cross Sections Based on an R-Matrix Analysis of the $^3$He System

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EVALUATION OF THE $d+t$ CROSS SECTIONS
BASED ON AN R-MATRIX ANALYSIS OF THE $^{5}$HE SYSTEM

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Arguments Against:

**Theory-based evaluation**

- Harder to do; takes longer.
- Limited ranges of mass, energy over which any one theory (model) is practical.
- Too much (approximate) theory $\Rightarrow$ bad representations of data.

**Curve-fitting evaluation**

- Completely unconstrained by physical principles.
- Burden falls entirely on measurements to get reliable results.
- Can get good representations of bad data.
ASYMPTOTIC REGION
(S matrix, phase shifts, etc.)

INTERIOR REGION
(Microscopic Calculations)

SURFACE

\[ R_{c'c} = (c'[H+\mathcal{L}-E]^{-1}c') = \sum_{\lambda} \frac{\gamma_{c'\lambda} \gamma_{c\lambda}}{E_{\lambda} - E} \]

- builds in fundamental conservation laws, symmetries, and analytic properties (causality, unitarity, etc.) of nuclear reactions.
- parametrizes only interior quantities (correct Coulomb, angular-momentum barrier penetration built in).
- explicit energy dependence (poles) ideal for describing resonances.
**Energy Dependent Analysis Code**

\[
R^{cc} = \sum_{\lambda} \frac{\gamma^c_{\lambda} \gamma^c_{\lambda}}{E_{\lambda} - E}
\]

- Calculate T- (or S-) matrix elements
- Form scattering observables using Wolfenstein trace formalism.
- Compare \(\chi^2\)
- Experimental data for all reactions
- Adjust parameters for minimum \(\chi^2\)

**Capabilities and Features**

1) Accomodates general (spins, masses, charges) two-body channels
2) Uses relativistic kinematics and R-matrix formulation
3) Calculates general scattering observables for 2 → 2 processes
4) Has rather general data-handling capabilities
5) Uses modified variable-metric search algorithm that gives parameter covariances at a solution.
\( {^5\text{He}} \) System Analysis

<table>
<thead>
<tr>
<th>Channel</th>
<th>( l_{\text{max}} )</th>
<th>( a_c ) (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d-t )</td>
<td>5</td>
<td>5.1</td>
</tr>
<tr>
<td>( n-4\text{He} )</td>
<td>5</td>
<td>3.0</td>
</tr>
<tr>
<td>( n-4\text{He}^* )</td>
<td>1</td>
<td>5.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Energy Range</th>
<th># Observable Types</th>
<th># Data Points</th>
<th>( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T(d,d)T )</td>
<td>( E_d=0-8 \text{ MeV} )</td>
<td>6</td>
<td>683</td>
<td>1284</td>
</tr>
<tr>
<td>( T(d,n)^4\text{He} )</td>
<td>( E_d=0-10 \text{ MeV} )</td>
<td>14</td>
<td>1241</td>
<td>1727</td>
</tr>
<tr>
<td>( T(d,n)^4\text{He}^* )</td>
<td>( E_d=4.8-8 \text{ MeV} )</td>
<td>1</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>( ^4\text{He}(n,n)^4\text{He} )</td>
<td>( E_n=0-28 \text{ MeV} )</td>
<td>2</td>
<td>813</td>
<td>1108</td>
</tr>
</tbody>
</table>

Totals: 23 2747 4134

\# parameters = 117 \( \Rightarrow \) \( \chi^2 \) per degree of freedom = 1.57

[109 phase parameters are necessary to describe the S matrix at a single energy]
\( T(d,d)T \) 197 MeV

\( T(d,d)T \) 3.97 MeV

\( T(d,d)T \) 6.10 MeV

\( T(d,d)T \) 8.00 MeV
## Renormalization Factors for T(d,n) Cross-Section Data

<table>
<thead>
<tr>
<th>Data set</th>
<th>Type</th>
<th>$E_d$ (MeV)</th>
<th>Scale factor</th>
<th>Scale error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jarrnie '84</td>
<td>$\sigma(E)$</td>
<td>0.008 - 0.070</td>
<td>1.017</td>
<td>1.26</td>
</tr>
<tr>
<td>Brown '87</td>
<td>$\sigma(E)$</td>
<td>0.053 - 0.116</td>
<td>1.025</td>
<td>- (rel.)</td>
</tr>
<tr>
<td>Bame '57</td>
<td>$\sigma(\theta)$</td>
<td>0.50</td>
<td>0.939</td>
<td>10</td>
</tr>
<tr>
<td>Bame '57</td>
<td>$\sigma(\theta)$</td>
<td>0.75</td>
<td>0.931</td>
<td>10</td>
</tr>
<tr>
<td>Bame '57</td>
<td>$\sigma(\theta)$</td>
<td>1.0</td>
<td>0.949</td>
<td>10</td>
</tr>
<tr>
<td>Bame '57</td>
<td>$\sigma(\theta)$</td>
<td>1.3</td>
<td>0.929</td>
<td>10</td>
</tr>
<tr>
<td>Bame '57</td>
<td>$\sigma(\theta)$</td>
<td>1.5</td>
<td>0.912</td>
<td>10</td>
</tr>
<tr>
<td>Bame '57</td>
<td>$\sigma(\theta)$</td>
<td>2.5</td>
<td>0.973</td>
<td>10</td>
</tr>
<tr>
<td>Bame '57</td>
<td>$\sigma(\theta)$</td>
<td>3.0</td>
<td>0.977</td>
<td>10</td>
</tr>
<tr>
<td>Bame '57</td>
<td>$\sigma(\theta)$</td>
<td>3.5</td>
<td>0.994</td>
<td>10</td>
</tr>
<tr>
<td>Bame '57</td>
<td>$\sigma(\theta)$</td>
<td>4.0</td>
<td>1.004</td>
<td>10</td>
</tr>
<tr>
<td>Bame '57</td>
<td>$\sigma(\theta)$</td>
<td>4.5</td>
<td>0.981</td>
<td>10</td>
</tr>
<tr>
<td>Bame '57</td>
<td>$\sigma(\theta)$</td>
<td>5.0</td>
<td>0.986</td>
<td>10</td>
</tr>
<tr>
<td>Bame '57</td>
<td>$\sigma(\theta)$</td>
<td>6.0</td>
<td>0.977</td>
<td>10</td>
</tr>
<tr>
<td>Bame '57</td>
<td>$\sigma(\theta)$</td>
<td>7.0</td>
<td>0.980</td>
<td>10</td>
</tr>
<tr>
<td>Paulsen '64</td>
<td>$\sigma(\theta)$</td>
<td>1.0</td>
<td>0.956</td>
<td>- (rel.)</td>
</tr>
<tr>
<td>Paulsen '64</td>
<td>$\sigma(\theta)$</td>
<td>3.0</td>
<td>0.974</td>
<td>- (rel.)</td>
</tr>
<tr>
<td>Ivarovich '68</td>
<td>$\sigma(\theta)$</td>
<td>4.2 - 10</td>
<td>1.016*</td>
<td>1.0</td>
</tr>
<tr>
<td>McDaniels '90**</td>
<td>$\sigma(\theta)$</td>
<td>5.0</td>
<td>0.996</td>
<td>1.5</td>
</tr>
<tr>
<td>McDaniels '90</td>
<td>$\sigma(\theta)$</td>
<td>6.0</td>
<td>0.980</td>
<td>1.5</td>
</tr>
<tr>
<td>Drosg '82</td>
<td>$\sigma(\theta)$</td>
<td>3.973</td>
<td>1.024</td>
<td>3.0</td>
</tr>
<tr>
<td>Drosg '78</td>
<td>$\sigma(\theta)$</td>
<td>7 - 10</td>
<td>0.986</td>
<td>1.5</td>
</tr>
<tr>
<td>Drosg '90</td>
<td>$\sigma(180^\circ)$</td>
<td>4.7 - 10</td>
<td>1.009</td>
<td>1.0</td>
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<tr>
<td>Goldberg '61</td>
<td>$\sigma(\theta)$</td>
<td>7.9</td>
<td>1.004</td>
<td>- (rel.)</td>
</tr>
</tbody>
</table>

* Experimental scale value of 1.028 ±0.01 determined by Drosg
** Based on McDaniels '73 as revised by Drosg
\( ^3\text{H}(d,n)^4\text{He} \) Cross Section

Scale Factor = 1.0173

Scale Uncertainty = ±1.26%

Los Alamos

ENDF/B-VI

Brown, 1987

Jarmie, 1984
$^{3}\text{H}(d,n)^{4}\text{He}$ Cross Section

$\sigma(E) \cdot E \exp[1.404/E^{1/2}]$

- Los Alamos
- ENDF/B-VI
- Conner, 1952
- Arnold, 1954
\(^3\text{H}(d,n)^4\text{He} \) Cross Section

\[
\sigma(E) \cdot E \cdot \exp[1.404/E^{1/2}]
\]

- Los Alamos
- ENDF/B-VI
- Kobzev, 1966
- Argo, 1952
- Balabanov, 1957

Deuteron Energy (MeV)
$^{3}\text{H}(d,n)^{4}\text{He}$ Cross Section

Los Alamos
ENDF/B-VI
Bame, 1957
Galonsky, 1956
Hemmendinger, 1955
Conner, 1952
Argo, 1952
Balabanov, 1957
Kobzev, 1966
$^3$H(d,n)$^4$He Cross Section

- Bame, 1957
- Magiera, 1975
- Drosd, 1978
- Goldberg, 1961
- McDaniel, 1973
- Stratton, 1952
- Galonsky, 1956

DEUTERON ENERGY (MeV)

CROSS SECTION (b)
\[ T(d,n)^{4}\text{He} \quad 5.00 \text{ MeV} \]

\[ T(d,n)^{4}\text{He} \quad 6.00 \text{ MeV} \]

\[ \theta_{\text{cm}} \]

\[ \frac{d\sigma}{d\theta} \]

McDaniels '73
$T(d,n)^4\text{He} \theta_{cm} = 0.0$

$T(d,n)^4\text{He} \theta_{cm} = 180.0$
Stability of Low-Energy Cross-Section Extrapolation

Present value of $\sigma_{d,n}(100 \text{ eV}) = 2.0506 \times 10^{-56} \text{ b}$

$\Rightarrow S(0) = 11.75 \text{ MeV-b}$

is:

5.5% higher than the value we had in 1979 (pre-Jarmie & Brown)

0.2% higher than the value we had in 1986 (used in Bosch & Hale)

0.5% higher than the value we had in 1992 (CSEWG 1993)
Conclusions

1. R-matrix theory, when used in its full multilevel, multichannel form, is an extremely useful tool for doing charged-particle evaluations for reactions in light systems at moderate energies.

2. No other evaluation for the d+t reactions has considered more data, and has been constrained by as much theory as the R-matrix calculations reported here. They give a good fit to all the data in the system, and especially for the T(d,n)\(^4\)He cross sections at energies below 10 MeV.

3. A single-level representation of the J\(^*\) = \(3/2^+\) R-matrix in the \(^5\)He system gives a cross-section extrapolation to low energies that is adequate to only about 5%. There are two other J\(^*\) = \(3/2^+\) resonances in the range below \(E_d = 8\) MeV, plus higher background-level contributions, that contribute 2.3% of the total reaction R-matrix element (or about 4.6% in the cross section) at low energies.

4. The calculations presented here allow a consistent evaluated file of cross sections and angular distributions to be constructed for all the d+t reactions [T(d,d), T(d,n), and T(d,n*)] at energies below 10 MeV, which give all the information necessary to do charged-particle transport.

5. The results of the R-matrix calculation at energies up to 10 MeV match well with the evaluation that has been done recently by Drosg for the T(d,n) cross section at higher energies.