EFFECTS OF TRANSMISSION LINES IN APPLICATIONS OF EXPLODING WIRES

R. Carroll Maninger
Librascope Division
General Precision, Inc.
Sunnyvale, California

In many applications, the lengths of connecting wires in exploding wire circuits may vary from a few inches to several feet. The connecting wires act as a transmission line with a characteristic impedance unique to the physical configuration of the circuit. If the time period of the wire explosion phenomenon is comparable to the transit time of electrical energy in the circuit, the efficiency of energy transfer to the wire is a function of the impedance mismatch between the circuit and the exploding wire. The effective impedance of the exploding wire is nonlinear; therefore, the impedance mismatch is also a function of time and the observed wire explosion phenomenon is affected by the circuit.

The effects of transmission line behavior of circuits on current, voltage, and energy transfer are treated in this paper by finding the transient solutions of the transmission line equations. In general cases, the solutions are multiple summation series requiring considerable numerical computation for comparison with experimental results. Fortunately, in most applications the solutions are simple staircase functions. These predict circuit behavior within a few percent of observations. A number of rules of thumb have been devised for use in estimating the degree of importance of transmission line effects in any given exploding wire circuit.

INTRODUCTION

The circuits most frequently used in the application and investigations of exploding wire phenomena are basically a series network of a charged capacitor, a fast discharge switch, connecting wires, and the exploding wire itself. The exploding wire phenomena occur in very short time periods ranging from a few nanoseconds to several microseconds at most. The exact durations of these time periods depend not only upon the nature of the exploding wire itself, but also upon the rate of electrical energy input to the wire. The rate of energy input is determined by the amount of initial stored energy in the system and by the circuit parameters. The associated electrical phenomena of the entire system, therefore, are of very short duration and must be treated as fast transient pulses.

It has long been recognized that these transients contain high-frequency components with significant amounts of energy and that small amounts of inductance can have appreciable effects on exploding wire phenomena. In addition to small inductors that may be intentionally used, there is always present the stray inductance of connecting wires, terminals, capacitor leads, etc. Because of the physical size of circuit components and the frequent necessity of locating the exploding wire some distance from the rest of the circuit, the various connecting wires will vary from a few inches to several feet in length. The stray inductance associated with the wiring can be appreciable. In the past, the stray inductance has been treated as a lumped parameter, which, together with circuit capacitance, produces a natural "ringing" frequency response in the system. In situations where the maximum physical dimensions of the entire system are only a few inches, this method of accounting for inductive effects in the circuit is valid.

However, as the physical dimensions of the system begin to expand beyond a few inches, the lumped-parameter approach to circuit analysis progressively deteriorates. In particular, if wiring lengths are a few feet, then the lumped-parameter approach will cause completely erroneous interpretations of exploding wire phenomena and the interrelated electrical effects. The basic reason for the breakdown of the lumped-parameter approach is that a small but finite transit time is required for electrical energy to propagate from one part of the circuit to another. This time increases as the dimensions of the circuit increase, and, when it becomes comparable to the time of growth and duration of the exploding wire phenomena, it can no longer be neglected. The effects of transit time in the circuit can be treated only by distributed-parameter methods. Thus, the longer parts of a circuit become waveguides or transmission lines. Under these conditions, the inductance of the wiring is considered in terms of inductance per unit length. The interwiring capacitance per unit length must also be considered, as well as the capacitance of the energy storage capacitor at the input to the network. In the general case the series resistance per unit length and shunt resistance per unit length should also be considered.

CIRCUIT ANALYSIS

Regardless of how long the wiring (transmission line) is, the voltage and current at any point x along it and at any time t are determined by the terminal conditions at the ends of the line and the partial differential equations [1]:

$$L\frac{\partial i}{\partial t} + Ri = -\frac{\partial e}{\partial x} \tag{1}$$

$$C\frac{\partial e}{\partial t} + Ge = -\frac{\partial i}{\partial x} \tag{2}$$

where L is the inductance per unit length of line, C is the capacitance per unit length of line, R is the resistance per unit length of line, G is the conductance per unit length of line, $i = i(x,t)$ is the current in the line at x and t, and $e = e(x,t)$ is the voltage in the line at x and t.

For most purposes, the wiring and transmission lines used in exploding wire work can be treated as lossless lines, that is, $R \approx 0$ because provision is made for carrying very high currents and $G \approx 0$ because good insulating materials are used between the wires to prevent high-voltage breakdown. With these approximations, equations (1) and (2) simplify to:

$$L\frac{\partial i}{\partial t} = -\frac{\partial e}{\partial x} \tag{3}$$

$$C\frac{\partial e}{\partial t} = -\frac{\partial i}{\partial x} \tag{4}$$

Terminal conditions for typical exploding wire circuits are shown in Fig. 1. The physical dimensions of C_1, L_1, L_2, R_1, R_2, and the switch are assumed to be small enough so that transit times through them are negligible. These components can then be considered as lumped parameters. The transmission line is assumed to have a geometrically uniform configuration over its entire length l. Regardless of the type and length of various possible configurations, each one will have a unique characteristic impedance Z_0 that

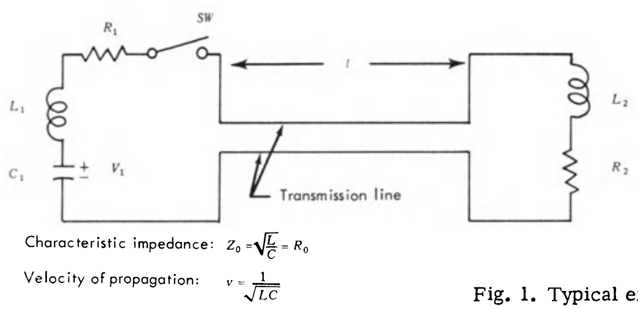
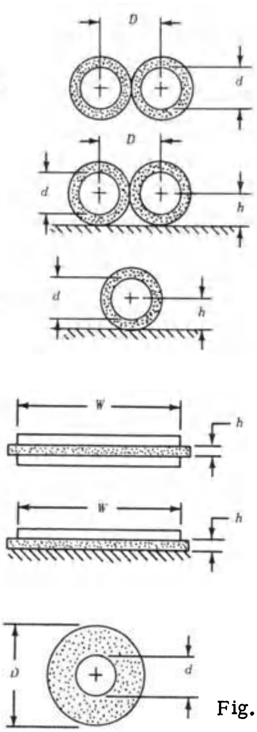


Fig. 1. Typical exploding wire circuit.



	Configuration	Z_0	Typical Z_0
(a)		$Z_0 = \frac{276}{\sqrt{\epsilon}} \log \frac{2D}{d}$	No. 16 Wire, 10-mil polyethylene insulation D = 0.070" - 82 ohms = 0.250" - 275 ohms = 1.500" - 500 ohms
(b)		$Z_0 = \frac{276}{\sqrt{\epsilon}} \log \left[\frac{2D}{d} \frac{1}{\sqrt{1 + \left(\frac{D}{2h}\right)^2}} \right]$	D = 0.070" - 54 ohms = 0.250" - 120 ohms = 1.50" - 330 ohms
(c)		$Z_0 = \frac{138}{\sqrt{\epsilon}} \log \frac{4h}{d}$	h = 0.035" - 41 ohms = 0.105" - 90 ohms = 0.250" - 180 ohms
(d)		$Z_0 = \frac{377}{\sqrt{\epsilon}} \frac{h}{w}$	Copper strip and insulating tape h = 0.010", W = 0.250" 10 ohms
(e)		$Z_0 = \frac{377}{\sqrt{\epsilon}} \frac{h}{W}$	h = 0.010", W = 0.250" 10 ohms
(f)		$Z_0 = \frac{138}{\sqrt{\epsilon}} \log \frac{D}{d}$	Commercial coaxial cable 50 - 72 ohms

Fig. 2. Typical wiring and transmission line configurations.

depends upon the cross-section geometry and the dielectric constant of the surrounding medium. Cross sections of several typical configurations are shown in Fig. 2 with the corresponding approximate equations [2] for Z_0 and some typical values for Z_0. Configuration (a) represents a pair of wires located many wire diameters away from metal surfaces. Configuration (b) represents a pair of wires on or close to a metal surface such as a chassis which does not carry current common with the wires. Configuration (c) represents a single wire close to a metal surface that serves as the return circuit for current through the wire. Configurations (d) and (e) are thin flat conductors analogous to (a) and (c), respectively. Configuration (f) is the common coaxial line.

Exact equations for Z_0 would take into account the fact that configurations (a), (b), and (c) have both solid insulation and air as a composite dielectric surrounding the conductors. However, for purposes of this paper, adequate approximations of Z_0 are obtained when "effective" values of ϵ are used; that is, when the wires touch, $\epsilon = \epsilon_{\text{insulation}}$; when the wires are separated many diameters, $\epsilon = \epsilon_{\text{air}}$. The important things to note in Fig. 2 are: (1) the configurations all have characteristic impedance greater than the resistances of most exploding wires reported in the literature; (2) circular configurations have higher Z_0 than flat configurations; and (3) the lowest values of Z_0 are obtained when the conductors are closest together.

The transit time t_p of energy propagation along the lines is independent of the configuration and is a function only of the dielectric constant ϵ in accordance with

$$t_p = \frac{1}{v} = \frac{\sqrt{\epsilon}}{c} \tag{5}$$

where v is the velocity of energy propagation in the transmission line and c is the velocity of light in vacuum. Values of various dielectric materials and the corresponding approximate transit times are shown in Table I.

Table I. Relative Dielectric Constants and Transit Times

Material	ϵ	t_p, ns/ft
Air	1.0	1.0
Teflon	2.1	1.4
Polyethylene	2.26	1.5
Lucite	3.0	1.7
Polystyrene	2.5	1.6
Vinyl chloride	4.0	2.0
Nylon	3.5	1.9
Glass	4-8	2-2.8
Rubber	3-6	1.7-2.4
Mycalex	7.5	2.7

The Laplace transforms I for the current $i(x,t)$ and E for the voltage $e(x,t)$ of the circuit in Fig. 1 are

$$I = \frac{V_1}{s(Z_0 + Z_1)} \frac{\exp[-nx] + N\exp[-n(2l-x)]}{1 - MN\exp[-2nl]} \tag{6}$$

$$E = \frac{V_1 Z_0}{s(Z_0 + Z_1)} \frac{\exp[-nx] - N\exp[-n(2l-x)]}{1 - MN\exp[-2nl]} \tag{7}$$

$$n = s\sqrt{LC}$$

$$M = \frac{Z_0 - Z_1}{Z_0 + Z_1}$$

$$N = \frac{Z_0 - Z_2}{Z_0 + Z_2}$$

s = complex *variable of transformation*

$$Z_1 = R_1 + L_1 s + \frac{1}{C_1 s}$$

$$Z_2 = R_2 + L_2 s$$

$$Z_0 = \sqrt{\frac{L}{C}} = R_0$$

When $x = l$, the solution for the current $i(t) = i(l,t)$ through the exploding wire (R_2, L_2) is given by

$$i(t) = \frac{2V_1 R_0}{L_1 L_2} \sum_{i=1}^{j} \sum_{j=1}^{K} \sum_{k=1}^{3} \frac{M_{ki}}{(j-i)!} \left[t - \frac{(2j-1)l}{v}\right]^{j-1}$$

$$\times \exp\left[s_k\left(t - \frac{(2j-1)l}{v}\right)\right] u\left(t - \frac{(2j-1)l}{v}\right)$$

$$M_{ki} = \frac{1}{(i-1)!} \left[\frac{d^{i-1}}{ds^{i-1}} \frac{(s-s_k)^j A_j(s)}{B_j(s)}\right]_{s=s_k}$$

$$A_j(s) = (s - r_1)^{j-1} (s - r_2)^{j-1} (s - r_3)^{j-1}$$

$$B_j(s) = (s - s_1)^j (s - s_2)^j (s - s_3)^j$$

$$r_1 = \frac{-(R_1 - R_0)}{2L_1} + \frac{1}{2}\left[\frac{(R_1 - R_0)^2}{L_1^2} - \frac{4}{L_1 C_1}\right]^{1/2} \tag{8}$$

$$r_2 = \frac{-(R_1 - R_0)}{2L_1} - \frac{1}{2}\left[\frac{(R_1 - R_0)^2}{L_1^2} - \frac{4}{L_1 C_1}\right]^{1/2}$$

$$r_3 = \frac{-(R_2 - R_0)}{L_2}$$

$$s_1 = \frac{-(R_1 + R_0)}{2L_1} + \frac{1}{2}\left[\frac{(R_1 + R_0)^2}{L_1^2} - \frac{4}{L_1C_1}\right]^{1/2}$$

$$s_2 = \frac{-(R_1 + R_0)}{2L_1} - \frac{1}{2}\left[\frac{(R_1 + R_0)^2}{L_1^2} - \frac{4}{L_1C_1}\right]^{1/2}$$

$$s_3 = \frac{-(R_2 + R_0)}{L_2}$$

A similar equation can be derived for the voltage across the exploding wire. The use of the above solution involves a considerable amount of computation for each specific case and is thus not convenient for illustrating the general effects of transmission lines in exploding wire circuits. It has been included here only to show how rapidly the analysis of a relatively simple circuit can become quite unwieldy mathematically.

The analysis is greatly simplified if the charged capacitor is replaced by a step function generator with an amplitude of V_1, and if $L_1 = L_2 = 0$. Experience has shown that these assumptions are valid for practical purposes of circuit design when

$$(R_1 + R_0)\, C_1 \geq \frac{5l}{v}(2K - 1) \tag{9}$$

$$\frac{L_1}{R_1 + R_0} \leq \frac{2l}{v} \tag{10}$$

$$\frac{L_2}{R_2 + R_0} \leq \frac{2l}{v} \tag{11}$$

where K is defined below in (14) and (18).

The Laplace transform for the current $i(t)$ through R_2 now becomes

$$I = \frac{V_1(1 + N)}{s(R_0 + R_1)} \frac{\exp[-nl]}{1 - MN \exp[-2nl]} \tag{12}$$

$$M = \frac{R_0 - R_1}{R_0 + R_1} \qquad N = \frac{R_0 - R_2}{R_0 + R_2}$$

where $MN \leq 1$, as is generally the case in exploding wire work,

$$\frac{1}{1 - MN \exp(-2nl)} = 1 + MN \exp(-2nl)$$

$$+ M^2N^2 \exp[-4nl] + \cdots + M^jN^j \exp[-2njl] \tag{13}$$

The transform may be written

$$I = \frac{V_1(1+N)}{s(R_0+R_1)} \sum_{j=1}^{K} M^{j-1} N^{j-1} \exp[-(2j-1)nl] \tag{14}$$

where $K = +\sqrt{\frac{vt}{2l} + \frac{1}{2}}$ is the number of reflections at R_2.

The solution $i(t)$ is given by

$$i(t) = \frac{V_1(1+N)}{R_0+R_1} \sum_{j=1}^{K} M^{j-1} N^{j-1}\, u\left(t - \frac{(2j-1)l}{v}\right) \tag{15}$$

$$u\left(t - \frac{(2j-1)l}{v}\right) = 0 \qquad t < \frac{(2j-1)l}{v}$$

$$= 1 \qquad t \geq \frac{(2j-1)l}{v}$$

This solution has the general form shown in Fig. 3. As pointed out in standard texts [1], this form can be interpreted as a superposition of waves reflected back and forth between the ends of the line. Upon each reflection at R_2, a portion of the energy is absorbed and the rest is reflected back to the source. The amount of energy absorbed (hence, the current) is a function of the factor MN and the total number of reflections preceding the one under consideration. The time between successive reflections at R_2 is solely a function of the length of the transmission line and the velocity of propagation along the line and is given by $2l/v$; this is the factor used to select the criteria for neglecting L_1 and L_2 in (10) and (11). The value i_m represents the maximum current possible in the circuit after an infinite number of reflections. In specific circuits, the time required to approach to within 90% of this quasi-steady-state condition is a measure of the accuracy of the assumption (9). If this time is considerably less than the time constant $(R_1 + R_0)C_1$ then

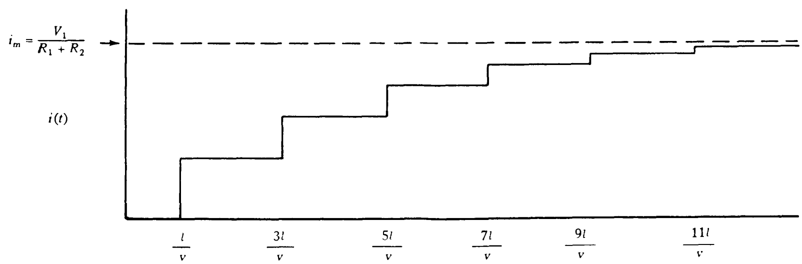


Fig. 3. Current through R_2 vs time.

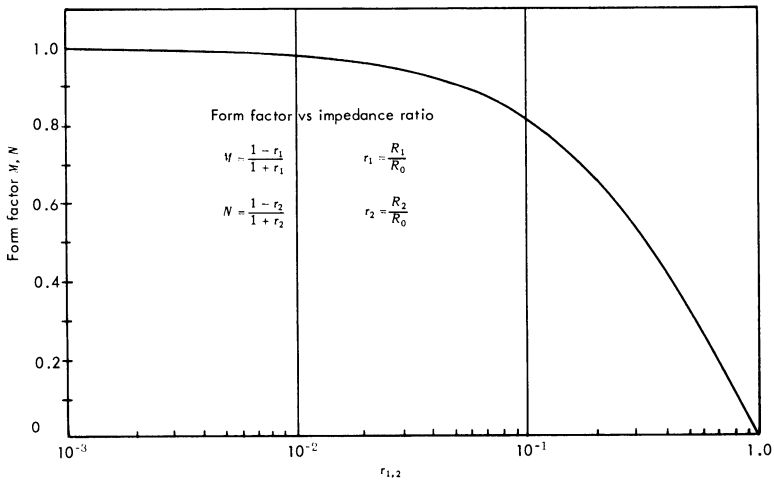


Fig. 4. Form factor vs impedance ratio.

the voltage on the capacitor cannot have dropped appreciably and the capacitor approximates a constant voltage source. After this quasi-steady state is reached, then of course the voltage on the capacitor will begin to drop and the solution for $i(t)$ will become progressively poorer with increasing time.

The dependence of the form factors M and N in (12), (13), (14), and (15) upon the ratio of termination impedances to the characteristic impedance of the transmission line are shown in Fig. 4. The significant feature to note is that for small values of impedance ratios, the values of M and N do not change much. Thus, even though the resistance R_2 of the exploding wire actually changes with time, the solution $i(t)$ is not greatly influenced. Ordinarily the maximum value of R_2 is used in applications of (15).

Since $u\left(t - \frac{(2j-1)l}{v}\right)$ is a unit step function, the summation in equation (15) is the sum of a finite geometric series for any given number of current steps, K, at R_2. The summation has the following properties:

$$S_K = \sum_{j=1}^{K} M^{j-1} N^{j-1} = \frac{1 - (MN)^K}{1 - MN} \tag{16}$$

$$\lim_{K \to \infty} S_K = \frac{1}{1 - MN} \tag{17}$$

$$K = \frac{vt}{2l} + \frac{1}{2} \text{ (to nearest integer since } t = 0) \tag{18}$$

Substituting S_K in (15) gives a more compact and convenient form for $i(t)$:

$$i(t) = \frac{V_1}{R_0 + R_1} (1 + N) S_K \tag{19}$$

Figure 5 shows S_K as a function of the product MN with the number of steps K as a parameter. Figure 6 shows S_K as a function of K with MN as the parameter. Inspection of these two figures shows that S_K, and hence $i(t)$, does not change rapidly with changes in MN when MN is small. This behavior, coupled with the insensitive behavior of N at high values with respect to R_2 mentioned above, further reduces the effects of a nonlinear R_2 on $i(t)$.

In the general case where $R_1 \neq R_0$ and $R_2 \neq R_0$, the maximum current i_m that can flow in R_2 is given by

$$i_m = \frac{V_1}{R_1 + R_2} \tag{20}$$

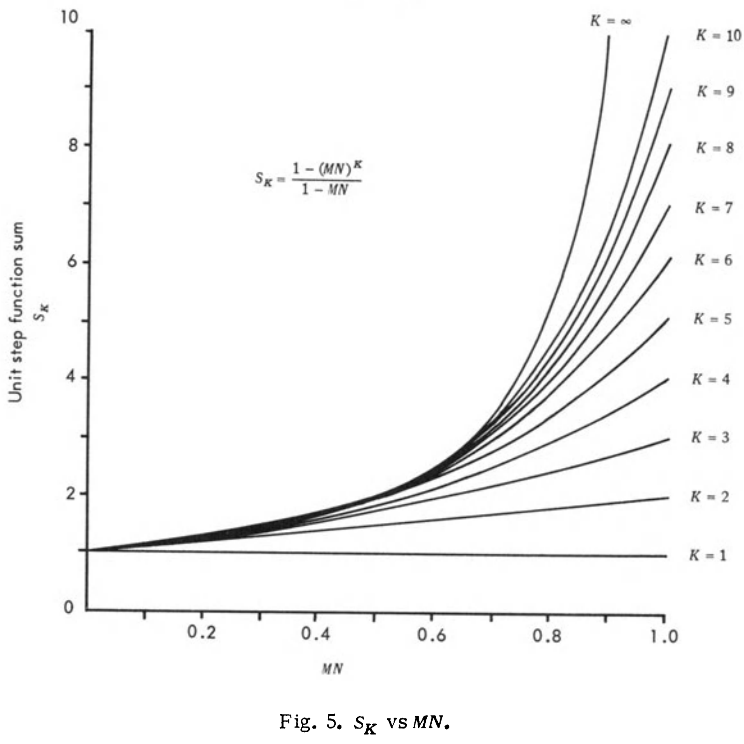


Fig. 5. S_K vs MN.

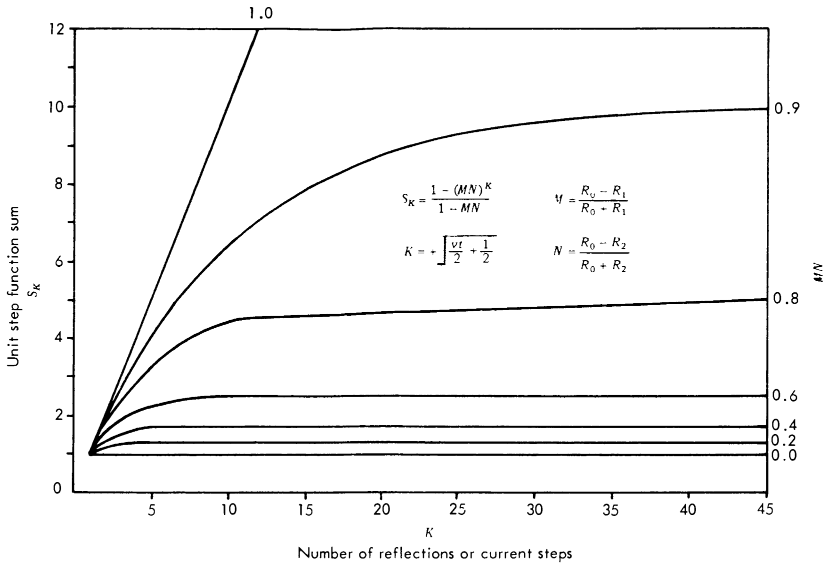


Fig. 6. Unit step function sum vs number of reflections.

Note that i_m is independent of $Z_0 = R_0$. Theoretically i_m is reached only after an infinite number of reflections at R_2. This means that the actual current approaches i_m asymptotically with time. However, $i(t)$ reaches specific fractions of i_m in finite times. This can be shown by taking the ratio of $i(t)$ and i_m. Thus, from (19) and (20):

$$\rho = \frac{i(t)}{i_m} = 1 - (MN)^K \tag{21}$$

where

$$\rho < 1$$

$$K = \frac{vt}{2l} + \frac{1}{2}$$

$$M = \frac{R_0 - R_1}{R_0 + R_1}$$

$$N = \frac{R_0 - R_2}{R_0 + R_2}$$

Equation (21) can be rewritten:

$$K(\rho) = \frac{\log(1-\rho)}{\log MN} \tag{22}$$

The value $K(\rho)$ is the number of reflections required for the current $i(t)$ to reach the fraction ρ of i_m. The relationship of the length

Table II. Number of Reflections to Reach 90% and 99% of Maximum Current and Length of Line to Reach 90% and 99% in 0.1 μsec

R_0, ohms	R_1, ohms	R_2, ohms	K, $\rho = 0.9$	K, $\rho = 0.99$	l_1, ft, $\rho = 0.9$	l_1, ft, $\rho = 0.99$	
500	1	0.1	520	1040	0.096	0.048	*
500	1	1.0	287	574	0.174	0.087	*
500	1	5.0	96	192	5.2	2.6	*
50	1	0.1	52	104	0.65	0.32	†
50	1	1.0	29	58	1.17	0.58	†
50	1	5.0	10	20	3.5	1.70	†
5	1	0.1	5	10	7.4	3.5	†
5	1	1.0	3	6	13.3	6.1	†
5	1	5.0	1	1	$(0 \to \infty)$	$(0 \to \infty)$	†

*Air dielectric.
†Polyethylene dielectric.

of line l, the velocity of propagation v, and the time t with $K(\rho)$ is

$$l = \frac{vt}{2K(\rho) - 1} \tag{23}$$

Table II shows the number of reflections (or current steps) required for $i(t)$ to reach 90% and 99% of the maximum theoretical current for several typical values of R_0, R_1, and R_2. The length of line l_1 for which $i/i_m = \rho$ is reached in 0.1 μsec is also given for each case. The first group of figures, for $R_0 = 500$, is typical of conditions for configurations (a) or (b) in Fig. 2 with $D \approx 1.5$ in. The second group, with $R_0 = 50$, is typical of configurations (a), (b), and (c) with close spacing and (f). The third group, with $R_0 = 5$, is typical of configurations (d) and (e) of Fig. 2.

Inspection of Table II shows that as the ratio of R_2 and R_0 decreases, the number of reflections required to reach a given fraction ρ increases. Furthermore, l_1 decreases with R_2/R_0. From (23) it is evident that if v is decreased by using dielectrics with higher dielectric constants than air or polyethylene, the critical lengths are further reduced.

In particular, it should be noted that the first two cases for $R_0 = 500$ reveal that wiring with lengths greater than 0.5 to 2 in. and with spacing of 1 to 1.5 in. has a significant effect for the first 0.1 μsec of the exploding wire phenomenon. Therefore, if the observed phenomenon lasts for 0.5 μsec or less, the results will be influenced more by the circuit than by the exploding wire. Such influence can be reduced by lowering the impedance R_0 of the wiring for any given length. Thus, care should be exercised in the layout of wiring to keep the conductors as close together as possible. For example, the first two cases for $R_0 = 50$ ohms show that the critical length becomes 4 to 12 in. instead of 0.5 to 2 in. If flat conductors are used to get $R_0 = 5$ ohms, then the critical length is 3 to 7 ft.

The last set of conditions for $R_0 = 5$ ohms is a special case where $R_0 = R_2$ and $R_0 \neq R_1$. Under this condition, the current flows through R_2 with just one step and no reflections occur; therefore, the length of the line has no effect on the current except to delay the position of the one current step with respect to the time of switch closure. The amplitude of the current step is given by (19) with $N = 0$; this value is also given by (20).

A second special case in which current is not affected by length of line except for time delays occurs when $R_1 = R_0$ and $R_2 \neq R_0$. In this case $N = 0$ and the amplitude of the current is also given by (20). This offers a convenient way to eliminate transmission line effects in exploding wire investigations and applications. It has a disadvantage, however, in that $R_1 = R_0$ usually has a much larger resistance value than the exploding wire and most of the stored energy of the circuit is dissipated in R_1 instead of the exploding wire. This inefficiency leads to capacitor sizes and voltage levels that may be prohibitive.

EXPERIMENTAL OBSERVATIONS

The results of a series of tests on transmission line effects are shown in Figs. 7 and 8. The basic circuit was the same as shown in Fig. 1. The same capacitor, switch, and current measuring circuit were used in all tests. In one group of tests, the transmission line was a 50-ohm line [RG58U(S)] 50 ft long, and R_2 was varied as indicated in Figs. 7 and 8. In the second group of tests, the transmission line was a 7.4-ohm line (special coaxial—Raychem type 52-007) 50 ft long; the terminating resistors were the same as above except when $R_2 = R_0$ (7.4 or 50.3). In both groups the exploding wire termination was a platinum wire with 0.002 in. diameter, $^3/_{32}$ in. length, and a cold dc resistance of 0.2 ohm. A coaxial shunt resistor of 0.0535 ohm was in series with all the terminating load resistances and made a part of R_2; the shunt resistor was used in conjunction with a Tektronix 517 oscilloscope to measure the load current.

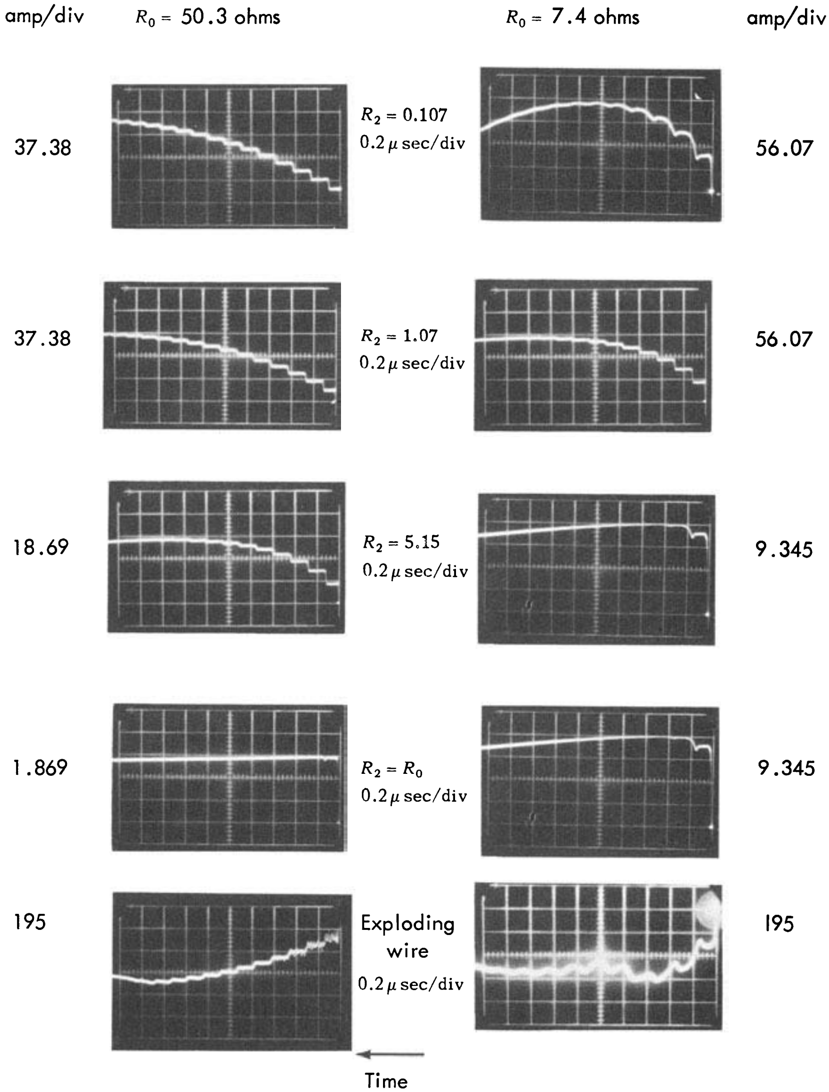


Fig. 7. Current vs time.

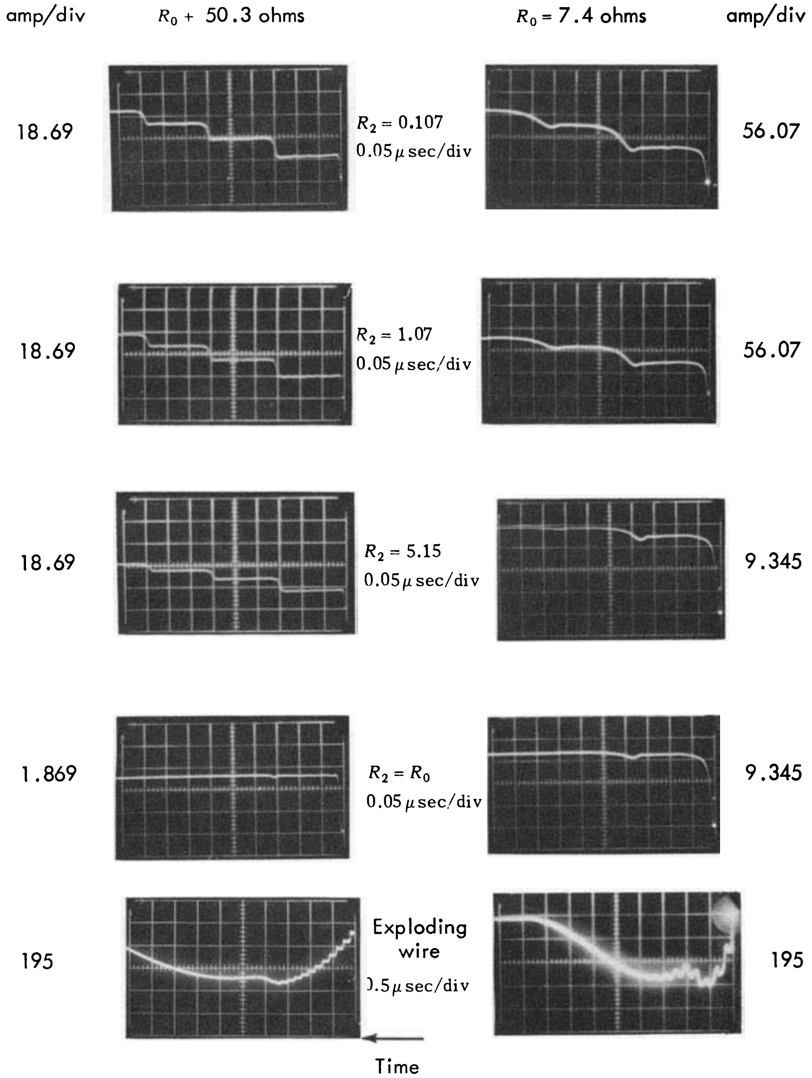


Fig. 8. Current vs time.

Table III. Experimental Circuit Parameters

C_1, μf	V_1, v	R_1*, ohms	L_1, μh	R_0, ohms	L_2, μh	R_2, ohms
1	336	1-4	≤ 0.54	50.3	<0.1	0.1071
1	336	1-4	≤ 0.54	50.3	<0.1	1.0665
1	336	1-4	≤ 0.54	50.3	<0.1	5.1535
1	270	1-4	≤ 0.54	50.3	<0.1	50.2735
1	2000	1-4	≤ 0.54	50.3	<0.1	Exploding wire
1	336	1-4	≤ 0.54	7.4	<0.1	0.1071
1	336	1-4	≤ 0.54	7.4	<0.1	1.0665
1	336	1-4	≤ 0.54	7.4	<0.1	5.1535
1	336	1-4	≤ 0.54	7.4	<0.1	7.3835
1	2000	1-4	≤ 0.54	7.4	<0.1	Exploding wire

* R_1 includes the effective resistance of the mercury switch, which varied from one test to another. The mean value was 2 ohms.

All of the current traces in Figs. 7 and 8 clearly show the staircase form as predicted in the above circuit analysis. (Note that the polarity of the exploding wire traces is reversed from the fixed resistance termination traces.) From the time duration of each step and the length of the line, it is evident that $1/v = 1.5$ ns/ft for the 50.3-ohm cable and $1/v = 1.8$ ns/ft for the 7.4-ohm cable.

When the criteria in (10) and (11) are applied for the circuit parameters in Table III, the results in Table IV are obtained. From these it may be seen that criteria (10) and (11) are satisfied, except that the margin of safety is considerably less for the 7.4-ohm cable than for the 50.3-ohm cable. When criterion (9) is applied, the results indicate that the effect of C_1 will be negligible for the first five to seven reflections in the 7.4-ohm cable, but thereafter the effect will become increasingly noticeable. This is readily apparent in Fig. 7 for $R_0 = 7.4$. However, when $R_0 = 50.3$, the effect of C_1 should not be appreciable until $K \geq 40$; the traces in Fig. 7 for $R_0 = 50.3$ show less than a 10% drop in 2 μsec.

Table IV. Criteria for Neglecting L_1 and L_2

R_0, ohms	$\frac{2l}{v}$, μsec	$\frac{L_1}{R_0+R_1}$, μsec	$\frac{L_2}{R_0+R_2}$, μsec
7.4	0.18	<0.058	<0.013
50.3	0.15	<0.01	<0.002

Table V. Number of Steps to Reach 0.9

R_0	R_2	$K(0.9)$ Calculated	$K(0.9)$ Observed
7.4	0.11	3-4	3-4
7.4	1.1	2-3	2-3
7.4	5.2	1-2	1-2
7.4	7.4	1	1
50.3	0.11	22-23	—
50.3	1.1	15-16	14-15
50.3	5.2	7-8	6-7
50.3	50.3	1	1

When (22) is used to find the number of steps required for current to reach 90% of its maximum value (20), the results in Table V are obtained. From these it is apparent that the agreement between theory and observation is good for finding the number of steps even though the use of criteria (9), (10), and (11) may be on the borderline.

One final observation—the experimental data show that the platinum wire exploded at $t = 1.8\ \mu$sec and $K = 12$ with the 50.3-ohm line and at $t = 0.8\ \mu$sec and $K = 4+$ with the 7.4-ohm line. This is direct evidence of the effects that transmission lines can have on exploding wire phenomena.

SUMMARY AND CONCLUSIONS

An analysis of transmission line effects in exploding wire circuits and criteria for its application have been developed. The analysis and criteria have been found to be adequate for order-of-magnitude calculations in actual circuits. The calculations show that the transmission line characteristics of physical exploding wire circuitry and wiring can have appreciable effects on exploding wire phenomena. Therefore, the transmission characteristics should be controlled and specified in each exploding wire experiment, not only for interpretation of results, but also for making it possible to duplicate experiments accurately.

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discussed in this paper, requires a great deal of skill and attention to elimination of ground loops and stray inductance in the measuring circuits themselves. All too frequently, ground loops and inductance in measuring circuits has confused the observed currents and voltages to such an extent that the true circuit behavior is completely hidden.

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