An object is thrown downward from the International Space Station (ISS) with a velocity of 1 m/sec. What is its subsequent orbit?

We assume for simplicity that the ISS is in a circular orbit about 400km above the surface of the earth, which means it is about 6800km from the center of the earth. The radius of the earth is about 6400km, so the acceleration of the earth's gravity at the location of the ISS is down by $(6400/6800)^2$ compared to its value on the surface of the earth, which is 9.8m/sec^2. So, at the ISS, the effective value of g is

$$g = 8.65 m/sec^2. \tag{1}$$

This in turn means that the orbital velocity is

$$v_0 = \sqrt{(8.65 m/sec^2) \times 6.8 \times 10^6 m} = 7700 m/sec. \tag{2}$$

Now consider any elliptical orbit around the earth, and let a 0-subscript refer to values at perigee, and a 1 subscript refer to apogee. The radius at the two locations is

$$r_0 = a(1-e), \qquad r_1 = a(1+e), \tag{3}$$

where a is the semimajor axis of the ellipse and e is the eccentricity. Let E and L refer to the energy and angular momentum per unit mass for the orbiting object. Evaluating these at perigee and apogee, we get

$$E = \frac{1}{2}v_0^2 - \frac{GM}{r_0} = \frac{1}{2}v_1^2 - \frac{GM}{r_1}, \tag{4}$$

where G is Newton's constant of gravitation and M is the mass of the earth, and we get

$$L = r_0 v_0 = r_1 v_1. \tag{5}$$

Now using (3) in (5) we get

$$L = a(1-e)v_0 = a(1+e)v_1, \tag{6}$$

which can be used to eliminate v_0 and v_1 from the energy equations (4). This gives

$$E = \frac{L^2}{2a^2(1-e)^2} - \frac{GM}{a(1-e)} = \frac{L^2}{2a^2(1+e)^2} - \frac{GM}{a(1+e)}. \tag{7}$$

Now subtracting these two expressions for the energy from each other and simplifying, we get

$$L^2 = GMa(1-e^2). \tag{8}$$

And plugging this back into one of the energy equations, we get

$$E = -\frac{GM}{2a}. \tag{9}$$

Notice that the total energy E is negative, because the orbits are bound. Equations (8) and (9) can be solved, if we want, to get a and e in terms of E and L.

Now let's return to the ISS, and change notation, using a 0 subscript henceforth to stand for values before the object was thrown down, and 1 subscript for after. Before our object is in the same circular orbit as the ISS, so $a_0 = 6800km$, $e_0 = 0$, $v_0 = 7700m/sec$.

After the object is thrown down, the kinetic energy per unit mass increases from

$$T_0 = \frac{1}{2}v_0^2 \tag{10}$$

to

$$T_1 = T_0 + \Delta T = T_0 + \frac{1}{2}(\Delta v)^2, \tag{11}$$

where $\Delta v = 1m/sec$. This is because the direction in which the object is thrown is orthogonal to the velocity vector of the ISS. The potential energy V does not change when the object is thrown. However, in a circular orbit, $V = -2T$, so the total energy (per unit mass) is initially

$$E_0 = T_0 + V_0 = -T_0, \tag{12}$$

and the final energy (per unit mass) is

$$E_1 = E_0 + \Delta T = -T_0 + \Delta T. \tag{13}$$

This means that

$$\frac{E_1}{E_0} = \frac{-T_0 + \Delta T}{-T_0} = 1 - \frac{\Delta T}{T_0} = 1 - \left(\frac{\Delta v}{v_0}\right)^2. \tag{14}$$

But from (9) we have

$$\frac{E_1}{E_0} = \frac{a_0}{a_1} = \frac{a_0}{a_0 + \Delta a} = 1 - \frac{\Delta a}{a_0}, \tag{15}$$

where Δa is the change in the semi-major axis of the orbit of the object after it is thrown and where we use the fact that $\Delta a/a_0$ is small. Combining this with (14) gives

$$\frac{\Delta a}{a_0} = \left(\frac{\Delta v}{v_0}\right)^2. \tag{16}$$

But $\Delta v/v_0 = 1/7700 = 1.3 \times 10^{-4}$, so

$$\Delta a = (1.3 \times 10^{-4})^2 \times 6800\text{km} = 14\text{cm}. \tag{17}$$

The semimajor axis of the orbit changes by only a very small amount after the object is thrown.

However, the orbit acquires a nonzero eccentricity. Since the angular momentum doesn't change when the object is thrown, (8) implies

$$L_0^2 = L_1^2 = GMa_0 = GMa_1(1 - e_1^2), \tag{18}$$

where we use $e_0 = 0$. This gives

$$\frac{a_0}{a_1} = 1 - \left(\frac{\Delta v}{v_0}\right)^2 = 1 - e_1^2, \tag{19}$$

or

$$e_1 = \frac{\Delta v}{v_0} = 1.3 \times 10^{-4}. \tag{20}$$

Ignoring the small change in the semimajor axis, this means that the new orbit has a perigee $e_1 a_0$ lower than the ISS orbit and an apogee the same amount higher. This amount is

$$e_1 a_0 = (1.3 \times 10^{-4}) \times 6800\text{km} = 890\text{m}. \tag{21}$$

This was the figure I quoted as 1km earlier, after a rough calculation. In any case, since ISS is well above the atmosphere, 890 meters closer to the earth will not bring our object signifiantly closer to the atmosphere.

This is not to deny that over a large number of orbits, atmospheric drag will slowly but eventually bring down both the ISS and the object thrown. To avoid that you need to be higher than 400km. But it is not because the object was thrown down with a velocity of 1m/sec relative to the ISS.

Note, by the way, that at the instant the object is thrown down from the ISS, its orbit is neither at apogee or perigee. Nevertheless, we know or can easily calculate the energy and angular momentum at this point, and since both are conserved around the entire orbit, they can be used to compute the distance to apogee and perigee of the new orbit, as was done.