Phys. Status Solidi A **206**, No. 7, 1410–1413 (2009) / **DOI** 10.1002/pssa.200925006

physica status solidi a
www.pss-a.com
applications and materials science

The role of residual stress in a Prince Rupert's drop of soda-lime glass undergoing a self-sustained and stable destruction/fracture wave

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Received 10 December 2008, revised 12 February 2009, accepted 24 February 2009
Published online 2 April 2009

PACS 46.40.Cd, 62.20.mm, 62.50.Ef, 81.40.Np, 87.63.L–

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It is shown experimentally using colour high-speed framing photography, carried out at 1 million frames per second that in a Prince Rupert's drop made of soda-lime glass, a self-sustained destruction/fracture wave can travel at a steady and stable speed of (1700 ± 100) m s^{-1}. It is also shown that this limiting velocity of the destruction/fracture wave, which is considerably smaller than the longitudinal wave velocity of ~5300 m s^{-1} in this glass, is due to crack bifurcation, which occurs in individual cracks moving at the limiting velocity, at the immediate front of the destruction/fracture wave. Moreover, it is also shown that in the intact part of a Prince Rupert's drop undergoing a self-sustained destruction/fracture wave, the residual stress state remains approximately unaltered until the intact part comes to within about one mm of the advancing destruction/fracture wave front.

1 Introduction It is well known that in a chemical explosive a self-sustained detonation wave can propagate at a stable and steady velocity [1]. The energy required for sustaining the detonation wave comes from the exothermic chemical reactions which occur in the reaction zone lying immediately behind the detonation front. About forty years ago, it was proposed by Galin and Cherepanov [2] and by Galin et al. [3] that by analogy with a detonation wave, a destruction/fracture wave (from here onwards we shall refer this to as fracture wave) can also propagate in an inert solid provided a sufficient amount of elastic potential energy is stored in the inert solid. This stored potential energy can come from an elastic compression or tension of the solid. Galin and Cherepanov [2] suggested that if in an elastically stressed solid a failure is initiated at some point, where a critical failure stress has been attained, the failure is accompanied by a release wave, which spreads through the compressed solid at the longitudinal wave velocity in the solid. Galin and Cherapanov[2] also suggested that the velocity of the fracture wave front is equal to the velocity of the longitudinal waves in the inert solid and that this velocity is not controlled by the velocity of the individual cracks in the solid. However, so far little clear experimental evidence for a self-sustained destruction/fracture wave and its relationship to the longitudinal waves in the solid has been provided. In this report we show, using very high-speed framing photography, that in specially prepared drops of soda-lime glass (usually referred to as Prince Rupert's drops, Batavian drops, Bologneser tears or drops, Bologneser Tränen (in German)), the fracture wave does indeed propagate in a self-sustained manner at a steady speed, which is considerably smaller than the speed of the longitudinal waves in the glass and is equal to the terminal crack velocity of individual cracks in the glass.

It may be noted that the recently reported observations from fracture waves in shock-compressed rods and blocks of Pyrex (a borosilicate glass) and soda-lime glass, although interesting in themselves, are not related to truly self-sustained fracture waves, since without the continuous support of the incident shock, generated by plate impact or by an explosive charge, there would be no fracture waves in the glass rods and blocks [4–6]. Moreover, some investigators have reported that the velocity of the fracture waves in shock-compressed glasses [5] and ceramics [7] decreases and may even stop propagating altogether with increasing distance of travel from the point of entry of the initial shock. These observations suggest that the fracture waves in shock-compressed solids are neither stable nor self-sustained because if they were self-sustained their velocity would remain independent of the distance of travel. They are not stable, because their velocity should tend to one value, independent of the experimental conditions. On the other hand, the fracture wave in a disintegrating Prince Rupert's drop is self-sustained, as no supporting shock is required. However, what is still unknown is how the residual stress within a disintegrating drop varies or not with time as the fracture wave front progresses from the tail of the drop towards its head. Therefore, it was the main purpose of this work to visualize qualitatively the residual stress state within a disintegrating Prince Rupert's drop and to use the information obtained for elucidating the mechanism of propagation of the self-sustained fracture wave in the drop.

2 Experimental We carried out our investigations of the fracture waves in Prince Rupert's drops, which were made of soda-lime glass. These drops were prepared by dropping red hot, molten blobs of the glass, formed on one end of a 10 mm diameter soda-lime glass rod by heating with a gas flame, into a 500 ml beaker of water at ~20 °C. The Rupert's drops thus formed were usually of a tadpole shape and their head diameters were usually about 6–8 mm. The tails of the drops were ~3–4 mm in diameter. When examined with polarized light, residual stresses were clearly seen within the drops, right from the tail to the head, even with the naked eye. Quite frequently, within the head region of the drops one or more vacuous bubbles were also formed. These bubbles would disappear on annealing at a sufficiently high temperature. Further details of the preparation method along with a detailed historical account of the investigations of the physical properties of these drops over the past 350 years or so can be found in Ref. [8]. It may be said here that the head of a Rupert's drop is so strong that it will not shatter when compressed to a load of 1500 kgf between hard tungsten carbide platens. On the other hand, their tail can be easily broken with just finger pressure. As soon as the tail breaks, the entire drop disintegrates explosively within a few microseconds, as shown in detail in Ref. [8].

The process of the formation of the drops results in the outer surface of the drops being in compression, and the interior of the drops in tension; both the compressive and tensile stresses are very high (compressive stress of 125 MPa), as reported earlier [8]. If a crack can be generated within the tension zone of a drop, the stress intensity at the crack tip will be very high and the entire drop will disintegrate rapidly by repeated crack bifurcation within a few microseconds. To study the propagation of the disintegration in detail, it is necessary to photograph the event at a framing speed of at least a million frames per second.

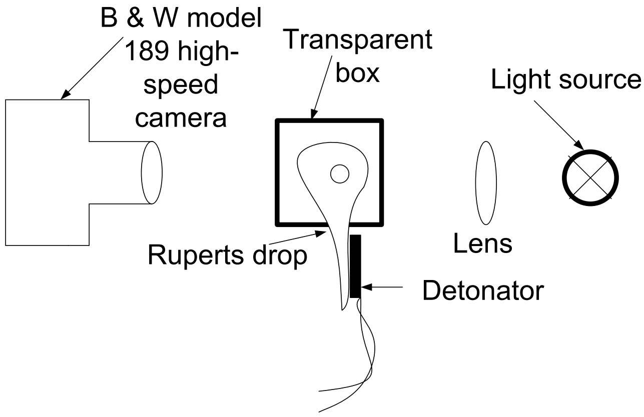

Figure 1 Schematic diagram of the experimental arrangement used for taking high-speed photographic sequences of the disintegration of Prince Rupert's drops using back lighting.

A schematic diagram of the experimental arrangement used is shown in Fig. 1. The test Rupert's drop was placed inside a transparent box made of polymethyl methacrylate, with part of the drop's tail lying outside the box. The box was then filled with glycerol (a clear viscous liquid of a matching refractive index), which considerably increased the visibility of the fracture wave front within the drop. To visualize the stress within a disintegrating Rupert's drop, a circular polarizer (not shown) was placed at each side of the box and in the path of the optical beam from the flash tube.

The disintegration of the test drop was initiated by the rapid explosion of a small fast acting detonator, which was placed on the tail of the drop, as shown in Fig. 1. The explosion of the detonator generated cracks within the tension zone of the tail of the drop from where the disintegration then started. The fracture wave front progressed from the tail to the head of the drop at a high speed and the propagation of the fracture wave front was photographed with a Beckman and Whitley model 189 rotating mirror camera (see Fig. 1). The event was backlit with the flash (duration of the flash was about 150 μs, but the intensity of the flash and its spectral distribution varied during this period) of a xenon-filled discharge tube (Q-arc Ltd, UK). Photographic sequences were taken at a 1× magnification on the photographic film (Fuji X-Tra 800 ASA). At a framing rate of one million frames per second, the exposure time per frame was 0.18 μs.

3 Results Selected frames from a typical sequence of high-speed photographs showing the propagation of a fracture wave in a Prince Rupert's drop are presented in Fig. 2. First, it may be noted that the fracture wave front divides the Rupert's drop into two distinct parts, one of which is comminuted (that is, extensively fragmented) and the other is intact. In every frame, except in frame number 12 in which the drop is completely comminuted, colour fringes within the intact part of the drop can be clearly seen. Each colour fringe is an isochromatic, which is the locus of

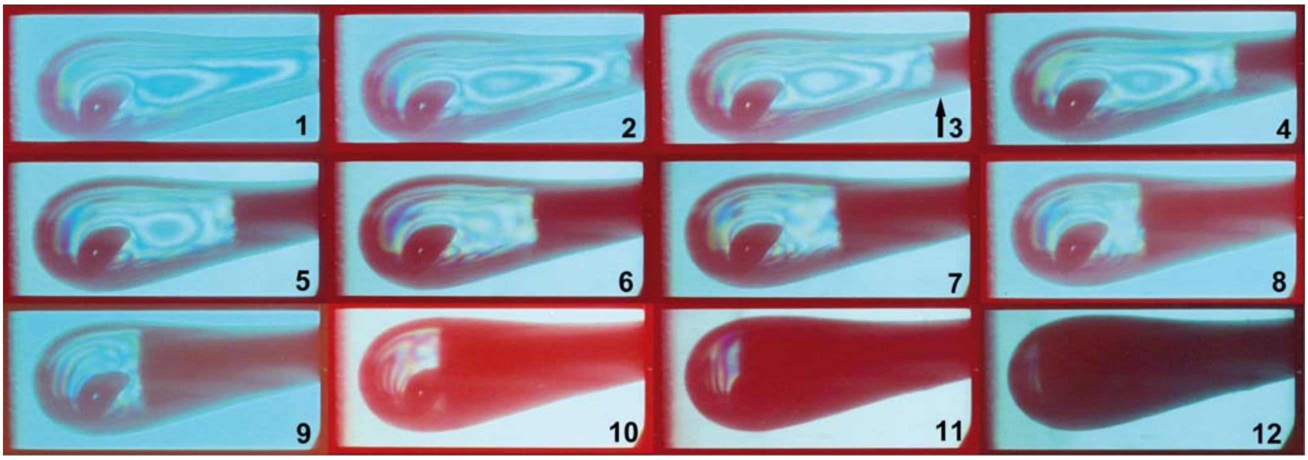

Figure 2 (online colour at: www.pss-a.com) Selected frames from a sequence of high-speed photographs, taken at a magnification of 1× on the photographic film, of the fracture wave in a Prince Rupert's drop of head diameter of 7.55 mm. The drop is immersed in glycerol whose refractive index is 1.47, which is close to the refractive index of soda-lime glass, which is 1.51. The dark disintegration front enters the field of view of the camera from the right hand side and propagates through the drop at a stable and steady speed of (1700 ± 100) m s^{-1}. The colour fringes within the intact part of the drop can be clearly seen. These fringes are isochromatics. Note that the fringe pattern does not alter significantly with the advancing fracture wave front. Inter-frame time is 1 μs and the exposure time per frame is 0.18 μs.

points within the drop at which the relative optical retardation is constant. The optical retardation is proportional to the difference between the two mutually perpendicular principal stresses in a plane normal to the optical beam. Frame 1 shows the drop about 10 μs after the detonator, attached to the tail of the drop, was exploded. The detonator was outside the field of view of the camera and it was about 15 mm from the right hand edge of the drop in this frame. In addition, a vacuous bubble can be seen within the head of the drop.

The explosion of the detonator causes an immediate (i.e., in less than 0.5 μs) initiation of a fracture wave in the drop and in frame 2, which is 12 μs after the initiation of fracture, the fracture wave enters the field of view from the right hand side. It should be noted that because the event is backlit, the fracture zone appears dark in the photographic sequences. In frame 3 the position of the fracture wave front is shown at the arrow. The remainder of the frames of the sequence show the propagation of the fracture wave towards the head of the drop at a uniform speed of (1700 ± 100) m s^{-1}. From this photographic sequence two further important observations may be made: (1) the overall shapes of the isochromatic lines at an intact region of the drop do not alter significantly until this region comes to within about one mm of the advancing fracture wave front, and (2) the isochromatics extend right to the fracture wave front. A selected frame from another photographic sequence, which shows this observation more clearly, is presented in Fig. 3. Further details of the drop are given in the caption of Fig. 3.

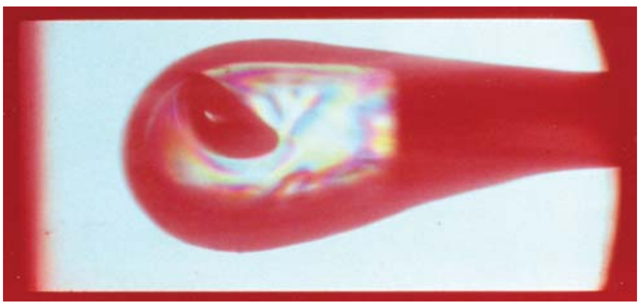

Figure 3 (online colour at: www.pss-a.com) A selected frame from a sequence of high-speed photographs, taken at a speed of one million frames per second, of the propagation of a fracture wave in a Prince Rupert's drop of soda-lime glass. Some of the isochromatics go right to the fracture wave front suggesting that the residual stress is not relaxed at the fracture wave front. The diameter of the head of the drop: 8 mm; exposure time of the frame: 0.18 μs.

4 Discussion Observations (1) and (2), described in the above, suggest that during the propagation of a self-sustained fracture wave in a Prince Rupert's drop the residual stress at a point within the intact part of the drop remains unaltered until it is just about to be overtaken by the advancing fracture wave front. In other words, during a short interval Δt of time the thickness of the region from which the energy required for the propagation of the fracture wave is $\Delta t \cdot c$, where c is the fracture wave speed. Note that $\Delta t \cdot c$ is also the distance by which the fracture wave front advances in the time interval Δt. Almost all the elastic strain energy in this zone goes into the fracture surface energy and kinetic energy of the fragments produced. In the photographic sequence shown in Fig. 2, the inter-frame time is just 1.0 μs, which is too short to record the movement of slow-moving fragments, 15 μm to 2.3 mm in size, which have velocities in the range 0–25 m s^{-1}. However, when the fragmentation process was photographed at a

Table 1 Fracture wave velocities and their characteristics in shock-compressed glasses.

investigators	fracture wave velocity (m s^{-1})	glass	comment	Ref.
Brar, Bless, and Rosenberg	2200 ± 200	soda-lime	constant velocity	[4]
Rasorenov, Kanel, Fortov, and Abasehov	2000 decreasing to 1000	K19 optical glass	fracture wave velocity decreases with increasing distance	[5]
Millett and Bourne	1650 to 2500	soda-lime	fracture wave velocity increases with increasing initial shock strength	[6]

much slower framing speed of 6000 frames per second or 110,000 frames per second [9], the flying fragments can be very clearly seen (see Fig. 9 of Ref. [8] and Fig. 2 of Ref. [9]). Furthermore, as shown earlier [10], the fracture wave front consists of a high number density of bifurcating cracks. Thus the speed of the self-sustained fracture wave in a Prince Rupert's drop is the same as the speed of the bifurcating cracks. Moreover, since crack bifurcation of individual cracks will occur when the cracks travel at their terminal velocities, we conclude that the speed of a self-sustained fracture wave in a Prince Rupert's drop is the same as the terminal velocity of individual cracks. In the case of individual cracks in plates of soda-lime glass, generated by small particle impact, speeds in the range 1500–1900 m s^{-1} have been measured [11, 12]. Thus it is suggested that the measured speed of (1700 ± 100) m s^{-1} of the self-sustained fracture wave in a Prince Rupert's drop is consistent with the above explanation. This fracture wave velocity is considerably smaller than the longitudinal wave velocity of 5300 m s^{-1} proposed by Galin and Cherepanov [2].

We have measured the fracture wave velocities in about twenty Rupert's drops of head diameters in the range 6–8 mm, which were placed in air, water or glycerol at room temperature. We did not find any effect of the atmosphere or the head diameter on the disintegration velocity, and the mean of all the measurements was (1660 ± 130) m s^{-1}.

It is interesting to compare the fracture wave velocity measured from a disintegrating Rupert's drop with the fracture wave velocities in shock compressed glasses. This comparison is shown in Table 1.

The data presented in Table 1 show that the fracture waves in soda-lime glass and K 19 optical glass show dependence on the experimental conditions. On the other hand, the fracture wave velocity in a Prince Rupert's drop is independent of the experimental conditions.

A comment should be made about the degree of roughness of the fracture wave front. As shown in Figs. 2 and 3, the roughness is less than about 0.5 mm, which is considerably smaller than that predicted recently by Grinfeld et al. [13]. Moreover, whereas Grinfeld et al. [12] predicted a finger-like shape of the fracture wave front, our high-speed photographic sequences show the fracture wave front to be quite smooth and flat.

5 Conclusions On the basis of our experimental observations given in the above, we conclude that in a Prince Rupert's drop, made of soda-lime glass, it is the residual stress which is the origin of the self-sustained fracture wave that can propagate in it. Moreover, the residual stress is not affected by any stress waves generated by the process of the disintegration of the drop during the propagation of the fracture wave. The fracture wave in the drop is stable and steady and the speed of the wave is equal to the terminal speed of individual cracks in the glass. This limiting speed is considerably smaller than the longitudinal wave speed in the glass. Finally, as a comparison, it may also be added that the origin and mechanisms of the fracture waves in shock-compressed glasses are still far from being clear [14, 15].

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